PRESERVICE PRIMARY TEACHERS' MENTAL COMPUTATION STRATEGIES IN STRUCTURALLY-RELATED ADDITION AND SUBTRACTION PROBLEMS

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ABSTRACT

PRESERVICE PRIMARY TEACHERS' MENTAL COMPUTATION STRATEGIES IN STRUCTURALLY-RELATED ADDITION AND SUBTRACTION PROBLEMS

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The purpose of this study is to analyze mental computation strategies used by preservice primary teachers while solving addition and subtraction problems that are related in terms of the part-part-whole structure. The data were gathered from 86 preservice primary teachers studying at a state university in Turkiye. In this single-case embedded design, a set of structurally-related two-digit addition and subtraction problems were implemented with semi-structured clinical interviews. The participants were given restricted time (5 seconds for each card) to answer the problems. After the whole set was completed, mental computation strategies were discussed in detail regarding the answers that the participants gave incorrectly and the answers they gave outside the given time. The interviews, checklists and researcher notes were used as main data source. The data was transcribed, the strategies were coded and analyzed according to the predetermined categories. The results of this study showed that preservice primary teachers do not have sufficient number sense and mental computation abilities. The participants expressed their need for paper and pen and used standard algorithm the most even though the problems contained shortcuts and were designed for using part part whole structure given in structurally-related problems. Besides the standard algorithm, benchmark, change both numbers, compensation, and part-part-whole were the other strategies that were used. On the other hand, some preservice primary teachers demonstrated unexpected strategies, and these strategies were named and described in this study. These results indicated that preservice primary teachers continue to apply algorithm related strategies and dependency on paper and pen from their early school lives. Considering that these participants will educate and guide their students in the future, it was thought vital for teacher candidates to strengthen their number sense and mental computation abilities.

Keywords: Number Sense, Mental Computation Strategies, Standard Algorithm, Strategy Development, Preservice Primary Teachers

ÖZ

SINIF ÖĞRETMENLİĞİ ÖĞRETMEN ADAYLARININ YAPISAL OLARAK İLİŞKİLİ TOPLAMA VE ÇIKARMA PROBLEMLERİNDEKİ ZİHİNDEN İŞLEM STRATEJİLERİ

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Bu çalışmanın amacı, sınıf öğretmeni adaylarının toplama ve çıkarma problemlerini çözerken kullandıkları zihinden işlem stratejilerini parça-parça-bütün yapısı açısından incelemektir. Veriler, Türkiye'de bir devlet üniversitesinde öğrenim gören 86 sınıf öğretmenliği öğretmen adayından toplanmıştır. Bu iç içe geçmiş tek durum deseninde, yapısal olarak ilişkili iki basamaklı toplama ve çıkarma problemleri yarı yapılandırılmış klinik görüşmeler ile uygulanmıştır. Katılımcılara soruları cevaplamaları için kısıtlı süre (her kart için 5 saniye) verilmiştir. Tüm set tamamlandıktan sonra sınıf öğretmenliği öğretmen adaylarının yanlış verdikleri ve verilen süre dışında verdikleri cevaplarına ilişkin zihinden işlem stratejileri ayrıntılı olarak tartışılmıştır. Görüşmeler, kontrol listeleri ve araştırmacı notları veri kaynağı olarak kullanılmıştır. Veriler yazıya dökülmüş, stratejiler kodlanmış ve önceden belirlenmiş kategorilere göre analiz edilmiştir. Bu çalışmanın sonucu, sınıf öğretmenliği öğretmen adaylarının sayı hissinin ve zihinden hesaplama becerilerinin yeterli olmadığını göstermiştir. Katılımcılar, problemler kısa yollar içermesine ve problemlerin tamamlayıcılarını kullanmak için tasarlanmış olmasına rağmen en çok standart algoritmayı kullanmış ve kâğıt kalem ihtiyaçlarını ifade etmiştir. Standart algoritmanın yanı sıra referans noktası, her iki sayıyı değiştirme, telafi ve parça-parça-bütün kullanılan diğer stratejilerdendir. Öte yandan, bazı sınıf öğretmenliği öğretmen adayları kendi stratejilerini geliştirmişler ve bu stratejiler isimlendirilerek açıklanmıştır. Bu çalışmanın sonuçları, sınıf öğretmenliği öğretmen adaylarının daha önceki okul yaşamlarından standart algoritma kullanma tercihlerini ve kâğıt kaleme bağlılıklarını devam ettiklerini göstermiştir. Bu katılımcıların gelecekte eğitim verecekleri ve öğrencilerine rehberlik edecekleri düşünüldüğünde, öğretmen adaylarının sayı duyusu ve zihinsel hesaplama becerilerini güçlendirmelerinin önemli olduğu düşünülmüştür.

Anahtar Kelimeler: Sayı Hissi, Zihinden İşlem Stratejileri, Standart Algoritma, Strateji Geliştirme, Sınıf Öğretmenliği Öğretmen Adayları

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ABBREVIATIONS

CCSSM	: Common Core State Standards for Mathematic
CESSM	: Curriculum and Evaluation Standards for School
D	: Difference
LD	: Large Distance
M	: Minuend
MoNE	: Ministry of National Education
NCTM	: National Council of Teachers of Mathematics
NRC	: National Research Council
PSSM	: Principles and Standards for School Mathematics
PST	: Preservice Teacher
S	: Subtrahend
SD	: Small Distance

CHAPTER 1

INTRODUCTION

Mathematics is a set of skills that individuals can use to handle the problems they encounter in daily life. However, new information and technologies are constantly changing and improvements affect the way we do math and communicate. Mathematics education needs to be revised in accordance with these improvements (National Research Council, 1989). Lester and Charles (1982) stated that the mathematics curriculum should focus on gaining the ability to cope with information that no one knows yet. They emphasized that the teachers cannot teach mathematics for an unknown future technology, but instead they should aim to develop thinking processes. As a natural result of this, while the importance of calculations with paper and pencil has decreased in mathematics education, skills such as estimation, problem solving, mental computation and reasoning have gained importance.

Considering these abilities, mental computation is one of the significant concepts that should be examined thoroughly and it is one of the critical components of number sense. Mental computation is the ability to perform precise mathematical calculations without the need of a calculator or any other device (Sowder, 1990). It is a crucial skill since it enables students to comprehend numbers properly, choose how to carry out operations, and develop calculation strategies (Varol & Farran, 2007). It was claimed that the focus should be on helping students create their own strategies by discovering, debating, and defending their ideas while teaching mental computation (Heirdsfield, 2011). When mental computation strategies are examined, it is seen that there are numerous strategies in the literature. These strategies were classified as counting and calculation strategies (Thompson, 1999), number sense and rule based strategies (Gülbağcı-Dede, 2015) or accumulative and replacement strategies (Olivier et al., 1990). Moreover, Thompson (1999, 2000) differentiated strategies as the addition and subtraction strategies up to 20 and addition and subtraction strategies for two digit numbers.

Even though there are many different mental computation strategies, the students need to discover these strategies by themselves, they should not be told to apply the steps because mathematics is not just a pile of formulas that need to be memorized (Carpenter et al., 1998; Threlfall, 2000). Mathematics has a structure that will require questioning, critical thinking, analysis, that is, the use of high-level cognitive skills. However, during the problem solving process, it is observed that students use memorized, written algorithms without thinking, focusing only on the result, and often find the wrong result (İymen, 2012; Şengül et al., 2012). A new focus on early numeracy is the creation of effective mental calculation skills based on invented strategies (McIntosh & Sparrow, 2004). When students' mathematical thinking is constrained by an overreliance on written procedures, it inhibits them from developing a number sense (Yang & Wu, 2010).

Mental computation and number sense are the two abilities that are interrelated, and these abilities affect each other and increase together. Since, number sense is one's comprehension of the meaning, representation, and relationships between numbers (National Council of Teacher of Mathematics (NCTM), 2000). Students with number sense can use numbers flexibly and fluently while making mental computation, make estimations and judgments about the size of numbers, question the validity of the outcomes of the computation, establish references regarding measurements and quantities, shift between different representations of numbers, and associate numbers, symbols, as well as operations (Adamuz-Povedano et al., 2021; Gersten & Chard, 1999; Greeno, 1991; Howden, 1989; Jordan et al., 2010; Markovits & Sowder, 1994; Olkun & Toluk-Uçar, 2018; Shumway, 2011).

NCTM (2000) released Principles and Standards for School Mathematics (PSSM) as a movement of revolution in mathematics education throughout the world. The main argument was which content will be given at which grade and the need for focusing content more deeply and showing the connections (Alsawaie, 2012; Hatfield et al., 2005; NCTM, 2000; Şengül & Gülbağcı-Dede, 2014). NCTM (2000) placed number sense as a major component of the core elementary mathematics curriculum. In addition, NCTM (2000) also remarked that students should achieve a rich understanding of numbers, which includes what the numbers are, how they can be represented with objects and number lines, how one number is related to another, and what the structures and properties of the numbers are. In this regard, Hatfield et al. (2005) noted that mathematical literacy and reasoning require number sense, and number sense is the crucial component of the core of the elementary mathematics curriculums.

In line with these purposes, reforms have been initiated in primary education mathematics teaching in some countries around the world. In the United States of America, sixteen standards have been determined by experts in order to increase the quality of the mathematics curriculum planned to be implemented in schools. Raising students who can use numbers in different situations in daily life, develop the ability to represent numbers in different structures, and have mental operation and estimation skills is determined as some of these standards (Rosenstein et al., 1996). The Curriculum and Evaluation Standards for School (CESSM) emphasized that a school's mathematics curriculum ought to have a primary objective as to teach the number sense. Additionally, the Number and Operations Standard of PSSM (NCTM, 2000) noted that the development of number sense is essential for these standards. In addition, although not directly, reflections of number sense have been observed in mathematics education curriculums in Turkiye. For example, one of the general aims of mathematics education is “Students will be able to use their estimation and mental computation skills effectively” (Ministry of National Education (MoNE), 2018, p. 9). For this purpose, although the importance of number sense is emphasized, sufficient acquisitions or activities were not included in the curriculum in terms of creating number sense (Umay et al., 2008).

In addition, the components of the number sense are investigated by several different researchers and these researchers stated the components as numbers, operations, understanding the problems, using strategies in these problems, number patterns, estimation, benchmarks and switching between possible different representations (Jordan et al., 2006; McIntosh et al., 1992; Resnick, 1989). According to these components, the abilities of individuals with number sense were also determined.

Developing part part whole relationships, making estimation, using benchmarks, producing mental computation strategies and computing flexibly are some of the abilities that can be observed in individuals with advanced number sense (Bresser & Holtzman, 1999; Greeno, 1991; Sood & Mackey, 2015). Şengül and Gülbağcı-Dede (2013) drew attention to the skills of students who have advanced number sense. As claimed by these researchers, by using number sense, students could think flexibly and produce different mental computation strategies for solving problems and use their knowledge of numbers and operations instead of memorizing the rules for solving problems and being dependent on paper-pencil.

In the literature, number sense and mental computation abilities of students has been the focus. However, studies developed to understand these abilities in students showed that number sense levels of students are insufficient, and students demonstrate their reliance on paper and pen and rule based strategies. Moreover, great importance is given to number sense and mental computation abilities in the mathematics curriculum. In regards to these results, teacher competencies are seen as important as the students to improve these abilities in students. In fact, Yang et al. (2009) stated that students' lack of number sense stem from the teachers' insufficiencies in number sense. Considering the above, both the changing understanding of education and the statements of international and national researchers show that new studies and new research should be carried out to examine the current state of number sense and mental computation abilities. In this study, preservice teachers are chosen as participants and their mental computation strategies are investigated. Since preservice teachers will most likely use their number sense abilities and mental strategies while solving problems when teaching mathematics to their students. For this reason, it is seen necessary to reveal the mental strategies of the preservice teachers, which are an indicator of whether they grasp the number sense correctly or not.

1.1 Purpose of the Study and Research Question

The main purpose of this study is to investigate the mental strategies of preservice primary teachers in solving structurally-related addition and subtraction problems (i.e.

the problems related in terms of part part whole structure).[a] In addition, the participants' strategic choices according to the characteristics of the problems is examined. Particularly, this research is implemented to understand the following research questions:

1. What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems?
   1.1. What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems within the allocated time?
   1.2. What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems outside the allocated time?
2. For not manageable problems within the allocated time,
   2.1. What are the strategies of preservice primary teachers produced outside the allocated time when solving structurally-related two-digit addition and subtraction problems?
   2.2. How do these strategies differ by year in the primary education program (i.e., freshman, sophomore, junior and senior)?
   2.3. How do these strategies differ by the characteristics of the structurally-related two-digit addition and subtraction problems?

1.2 Significance of the Study

Mathematics is important not only for students but also for every member of society because everyone should develop mathematical thinking skills and be able to use these skills when necessary. Mathematics is a part of our daily lives and we can only adapt to the ever-changing and developing world with mathematical skills. Noticing

[a] There are different types of structurally-related problems such as Decoy, Far, Near (Category 1), Small Distance, Large Distance (Category 2), Subtraction, Addition (Category 3). The main characteristics of these problems is that when a problem is given and presents part part whole structure (e.g. 31-28=3), another problem asks for one component in that structure (e.g. 28+3=?).

mathematical relationships, developing different strategies for solving problems, estimating, making inferences, and making generalizations are skills that every person could use to do mathematics effectively and flexibly. In accordance with this, number sense has been the focus of research because it is the ability which has an important effect on individuals as they learn new concepts. It is defined as making logical inferences about wide applications of numbers, identifying mathematics misconceptions, selecting the most efficient method of calculation, and recognizing numerical patterns (Hope, 1989). As Jordan et al. (2010) indicated advanced number sense knowledge is essential and linked to students’ future mathematical capabilities and competencies.

Considering the number sense components and indicators of number sense, it was seen the mental computation abilities also needed to be investigated. Mental computation should be an indispensable part of mathematics education as it is in our daily lives. For years, written algorithms were seen important in mathematics education and then they started to be perceived as insufficient for the needs of individuals in solving daily life problems, so to meet these requirements, mental calculation and estimations were mostly preferred. Recent studies in this area have shown that the process of mental computation should be emphasized. It is vital for students to learn mental computation skills and produce invented strategies.

There are several studies to assess the students' use of number sense and mental computation abilities, and majority of the research showed low level of number sense abilities in students. Yang et al. (2009) pointed out that the reason for this failure in students is the lack of number sense in teachers, as well as their inability to develop students' number sense. Therefore, teacher competencies are also very important in order to develop number sense of students. Teachers may choose more intelligently the approaches they want to encourage if they are aware of the strategies (Fuson et al., 1997). Elementary school mathematics classrooms foster or promote the development of unique notions of numbers through the language used by teachers and students, the type of physical manipulatives, the problems to be answered, and the structured class activities. Together, these components could assist students in developing values for numbers (Fuson, 1997). Since a teacher who does not have mental computation skills would not be aware of the methods, they would teach with traditional written algorithms and would not be able to guide their students to develop mental strategies. In order for students to gain mental computation skills, the teacher education has gained importance. If students and teachers are aware of mental computation connections, the acquisition of mental operation skills would occupy an important place in the classroom. Instead of a mathematics education that encourages memorizing the inferences and solutions, students should be offered learning environments that allow them to be active in their learning and produce their own solutions and strategies. According to Korkmaz and Gur (2006), primary and elementary mathematics teachers are the first ones who should have competencies such as using and applying mathematical knowledge and skills, critical thinking, questioning, doing mathematics, and problem solving.

There are plenty of the studies in the literature revealed that both primary and elementary students (Alsawaie, 2012; Kayhan-Altay, 2010; Reys et al., 1999; Reys & Yang, 1998; Yang, 2005) and primary school teacher candidates have low level of number sense (Yaman, 2015; Yang et al., 2009; Şengül, 2013). In addition to these studies, researcher have focused on mental computation abilities and strategies. These researchers have found out that participants chose standard algorithm dominantly as opposed to invented strategies (Beishuezen et al., 1997; Carroll, 2000; Güç & Karadeniz, 2016; Kabaran & Işık-Tertemiz, 2019; Torbeyns et al., 2008; Torbeyns & Verschaffel, 2016; Yang & Huang, 2014). Interestingly, some studies showed high accuracy with standard algorithm (Carroll, 2000; Torbeyns & Verschaffel, 2013; Torbeyns & Verschaffel, 2016) whereas other studies demonstrated low achievement with standard algorithm (Kabaran & Işık-Tertemiz, 2019; Yang & Huang, 2014).

When the studies related with the mental computation strategies are investigated, it is seen that only a few studies have examined one's usage of mental strategies for multi digit addition and subtraction while the majority of research focuses on single-digit addition and subtraction (Peters et al., 2010). Differently, this study is carried out with structurally-related two digit addition and subtraction problems which are defined below. Moreover, majority of the studies have focused on applying a number sense test and investigating the mental computation abilities according to the number sense levels. This study diverges from other studies regarding its design. The problems consisted part-part-whole and complement relationships and these problems were prepared based on two different researches (Paliwal & Baroody, 2020; Peters et al., 2010). For example, the problems were asking 28+3=? after giving 31-28=3 and there were different categories according to the characteristics of the problems (i.e. decoy, near complement and far complement; small distance and large distance; subtraction and addition).

While most of the research has focused on children, very few have worked with preservice teachers. It is inferred that preservice primary teachers' mental computation abilities and their strategies are vital since they are the ones who will teach and guide their students. Furthermore, their abilities need to be discovered with distinct methods. Also, it is very important in our country to determine the profiles of primary school teachers, who have the most important place in the development of students' number sense. Therefore, the purpose of this study is to investigate how preservice primary teachers employ mental strategies in part part whole related two-digit addition and subtraction problems. This study will contribute to the literature because it was studied with preservice primary teachers who are a less focused group, employed a different design than other studies, and a data collection tool that has not been studied before in Turkiye context.

1.3 Definition of Important Terms

The following important terms are associated with the study and definitions are given in this section.

Number sense: A person's basic comprehension of numbers and operations, as well as their ability and tendency to apply that understanding in a variety of ways to make mathematical judgements and devise helpful strategies for dealing with numbers and operations. Furthermore, it is characterized as a good intuition for numbers and flexible use of the relationship between numbers and operation (Howden, 1989; McIntosh et al., 1992; Reys et al., 1999).

Mental Computation: The procedure of doing arithmetical calculations without the use of external instruments is known as mental computation (Sowder, 1990). Particularly, mental calculations that are often conducted "in the mind" rather than "on paper" (Harries & Spooner, 2000, p. 75).

Strategy: The art of creating or applying plans or methods carefully to achieve an objective (Merriam-Webster, n.d.).

Mental Computation Strategies: Mental computation strategies are clever ways to compute that based on a person's fundamental knowledge of the number system and arithmetic operations, as well as advanced sense of numbers and comprehension of the fundamental number facts (Torbeyns & Verschaffel, 2015).

Part-Part-Whole Structure: There are three different quantities in each problem, which are the start, the change, and the result. In this structure, start and change are the parts, and the result is the whole. For example, in 4+8=12, 4 and 8 are both parts of the total of 12. Any of these three quantities could be unknown in a problem (Stocker Jr., 2021). In this study, when a problem is given (e.g. 31-28=3) and presented part part whole structure, another problem asks for one component in that structure (e.g. 28+3=?).

Structurally-Related Addition and Subtraction Problems: This study includes a variety of structurally-related problems, including decoy, far, near (Category 1), small distance, large distance (Category 2), subtraction, and addition (Category 3). Near Complement problems are the problems where the second addend is subtracted from the sum. For example, 3+8=11, 11-8=? is a near complement problem. Far Complement problems, on the other hand, are those in which the first addend is subtracted from the sum (e.g., 7+6=13, 13-7=?). Lastly, problems involving the same numbers in addition and subtraction but distinct part-whole relationships are described as decoy problems (e.g., 17 + 8 = 25, 17 – 8 =?). Large distance problems are where the difference between subtrahend (S) and difference (D) is bigger than ten, and small distance problems are where the difference between subtrahend (S) and difference (D) is less than ten. For example, 34-8=26, 26+8=? is a large distance problem since 26-8=14 and it is bigger than ten. Also, 43-18=25, 25+18=? is a small distance problem because 25-18=7 and it is smaller than ten (Peters et al., 2010). The first operations are presented in either addition or subtraction format, and these problems are in the third category. Therefore, one problem had one of the characteristics under each category (e.g., 34-8=26, 26+8=? is a near complement, large distance, subtraction problem) and these problems are called as structurally-related addition and subtraction problems.

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to analyze the mental computation strategies used by preservice primary teachers while solving two-digit addition and subtraction problems that are related in terms of the part-part-whole structure. Also, the participants' strategic choices according to the characteristics of the problems was investigated. This chapter presents the number sense concept, addition and subtraction operations, and mental computation concept. Firstly, the concept of number sense, components of number sense and the indicators of number sense will be examined thoroughly. Secondly, the addition and subtraction operations, the structures of addition and subtraction problems, operation sense concept and mathematical proficiency will be studied comprehensively. Finally, the nature of the mental computation strategies and different mental computation strategies will be investigated in detail.

2.1 The Concept of Number Sense

In 1989, number sense was originally mentioned at a NCTM conference. Before that Crowter (1959) used the term "numeracy " to refer to the present principles behind the idea of number sense in a similar way (p. 270). Besides, number sense has been perceived as one of the most important components of mathematical literacy and reasoning (Sheffield & Cruikshank, 2005). When the literature is examined, it is seen that there are several different structures and definitions of the number sense. In this regard, Case (1998) stated that "number sense is difficult to define but easy to recognize" (p. 1). Gersten et al. (2005) presented this situation as there cannot be two researchers who define the number sense in exactly the same way.

In the literature, it is also possible to come across studies conducted by neuropsychologists and mathematics educators on the origin of the number sense

(Dehaene, 1997; Greeno, 1991; Howden, 1989; Lipton & Spelke, 2003). There are different opinions put forward in these studies regarding the origin of the number sense. Neuropsychologists claimed that people have a number sense, just like the sense of color, and were born with these senses (Dehaene, 1997; Lipton & Spelke, 2003). Dehaene (1997) also claimed that there are "numerate neurons" in the brain of people that instinctively perceives numbers, and the calculations made are all caused by the activation of neuron cells (p. 228). Despite this opinion, which claimed that the number sense is a biological hardware that is completely related to the structure of the brain, another opinion is that the number sense was considered by mathematics educators as more of a knowledge and skill than an internal process. According to this view, which was mostly adopted by mathematics educators, the number sense is not something that is stable and unchangeable, it is an ability that can be learned, improved and expanded with age (Dehaene, 1997; Greeno, 1991; Lipton & Spelke, 2003; McIntosh et al., 1992).

According to McIntosh et al. (1992), number sense is a person's basic comprehension of numbers and operations, as well as their ability and tendency to apply that understanding in a variety of ways to make mathematical judgements and devise helpful strategies for dealing with numbers and operations. Similarly, according to another definition, the number sense means all the relationships of the numbers rather than knowing the number which is the ability of relating few-many, part-whole, their relations with real quantities and their measurements in the environment. It was also added to this definition that number sense is the ability of making sense (Olkun & Toluk-Uçar, 2018).

2.1.1 The Components of Number Sense

Due to its difficulty in defining, the number sense has generated arguments about the number sense components (Er & Dinç-Artut, 2022). Berch (2005) developed a list of 30 different components of number sense by examining some other studies about number sense. Similarly, in a study for examining the mathematical structure of number sense and the components that was conducted by Politylo et al. (2011), 40 studies related to number sense were examined and it was revealed that 34 different number sense components were used in the studies in the literature.

As a result of investigations, number sense has been shown to be comprised of mental number line, knowledge, awareness, desire, process, skill, intuition, feel, recognition, ability, expectation, and conceptual framework (Berch, 2000). In their investigation, Şengül and Gülbağcı-Dede (2013) have looked into the classifications of number sense components and found that there was no standard classification for number sense components in the literature. They have claimed that McIntosh et al. (1992) have created the most thorough classification and the conceptual framework has three main components for number sense: numbers, operations, and applications of operations with numbers. McIntosh et al. (1992) have mentioned representations of numbers, order of numbers, the magnitude of numbers, multiple representations of numbers, decomposition, recomposition, benchmarks, relationships among operations, understanding problems, using different strategies and reasoning the final result under these main components.

Where Jordan et al. (2006) have classified different components under five areas: counting, number knowledge, number transformation, estimation, and number patterns, Lago and DiPerne (2010) have determined counting aloud, measuring concepts, non-verbal calculation, number determination, noticing the quantity. On the other hand, Reys et al. (1999) have determined six components of number sense: understanding the meaning and magnitude of the number, understanding and using representations of a number, understanding the meaning and effect of operations, the use and meaning of synonym expressions, counting and flexible trading strategies for mental trading and measurement references. Furthermore, Faulkner and Cain (2009) have created a number sense framework with the components such as quantity/magnitude, base ten, equality, forms of number, numeration, proportional reasoning and algebraic and geometric thinking.

In contrast to other classifications, Resnick (1989) has listed the possible indicators of number sense and it is especially noted that these are not components of the number sense. However, these indicators were relationships among numbers, reasoning the result, estimation, reassembling the numbers, and switching between possible different representations in a flexible way. Similarly, Yang (2003) called these components as characteristics of the number sense instead of components in the conceptual framework. According to the researcher, the number sense has five characteristics: understanding the meaning of the number, understanding the size of numbers, proper use of measurement references, understanding the relative effects of operations on numbers and developing different strategies appropriately and judging the reasonableness of the answers.

The Table 2.1. below shows a summary of the components of the number sense determined by aforementioned researchers.

Table 2.1 The Components of Number Sense

Researcher	**Components**
Berch (2000)	Mental number line, knowledge, awareness, desire, process, skill, intuition, feel, recognition, ability, expectation, and conceptual framework
McIntosh et al. (1992)	Numbers · Number order, place value, multiple representations of numbers, decomposition, recomposition, benchmarks, the magnitude of numbers. Operations · Understanding the effect of operations, mathematical properties, relationships among operations. Applications of Operations with Numbers · Understanding problems, using different strategies and reasoning the final result under these main components.
Jordan et al. (2006)	Counting, number knowledge, number transformation, estimation, and number patterns
Lago & DiPerne (2010)	Counting aloud, measuring concepts, non-verbal calculation, number determination, noticing the quantity.
Reys et al. (1999)	Understanding the meaning and magnitude of the number, understanding and using representations of a number, understanding the meaning and effect of operations, the use and meaning of synonym expressions, counting and flexible trading strategies for mental trading and measurement references.
Faulkner & Cain (2009)	Quantity/magnitude, base ten, equality, forms of number, numeration, proportional reasoning and algebraic and geometric thinking.
Resnick (1989)	Relationships among numbers, reasoning the result, estimation, reassembling the numbers, and switching between possible different representations in a flexible way.
Yang (2003)	Understanding the meaning of the number, understanding the size of numbers, proper use of measurement references, understanding the relative effects of operations on numbers and developing different strategies appropriately and judging the reasonableness of the answers.

As seen in Table 2.1. there is not a standard components list; however, it was observed that the researchers mentioned some common components such as number knowledge, relationship among numbers and operations, benchmarks, estimation and flexible use of mental strategies.

2.1.2 The Indicators of Number Sense

According to Sood and Mackey (2015), students could develop four types of number relationships in the early number sense phase: spatial relationship, one and two more, one and two less relationships, anchors or benchmarks of 5 and 10, part-part-whole relationships. Students with developed spatial relationships can learn to identify groups of items in patterned formations without having to count them. Furthermore, one and two more, one and two less relationships is building the connection between 6 and 8 as two more than and two less than. The benchmarks of 5 and 10 are related to other numbers, specifically 5 and 10. Since the benchmark number 10 is made up of two 5s and when forming links between numbers smaller than 10 both 5 and 10 are effective anchors to apply (Sood & Jitendra, 2013). This link not only enables students to consider numerous number combinations, but it also helps them develop mental computation skills for higher numbers (Van de Walle et al, 2013). Part-part-whole relationship is about either constructing the whole with two or more parts, or decomposing the whole into two or more parts (Fischer, 1990).

In CESSM (NCTM, 1989), the characteristics of individuals with number sense was explained as the ability to understand the meanings of numbers well, develop multiple bonds between numbers, recognize the relative magnitudes of numbers, understand the power of operations on numbers, develop a reference benchmark point for measurements of surrounding objects. Flexible mental computation, numerical estimation, and quantitative judgment are all examples of number sense, which is a collection of vital yet elusive capabilities (Greeno, 1991).

Number sense is an intuitive understanding of numbers and their relationships, and it emerges over time as a result of experimenting with numbers, seeing them in different situations, and linking them in ways that are not constrained by traditional algorithms

(Howden, 1989). In addition to numbers and understanding in general, number sense understanding makes use of efficient and appropriate methods to design training and ability to produce effective strategies to manage (Reys et al., 1999). Berch (2005) revealed that individuals with a number sense can achieve establishing connections for using numerical strategies, producing strategies for solving complicated math problems and creating numeric calculations. An individual who uses number sense strategies can round, separate, or combine numbers, use number isomorphic or reference points, relate numbers and operations, predict the outcome of the operation and do similar operations (Gülbağcı-Dede, 2015).

Howden (1989) defined number sense as the ability to discern that there are different ways to arrive at a solution by making inferences that would make sense, rather than just certain rules to be followed. Hatfield et al. (2005) added that number sense provides the ability to combine numbers that recognize 8 as 5 and 3 so that the individual does not need to memorize 5+3=8 or 8-5=3. Students with number sense ability can understand the relationships among numbers, the effects of operations and be aware of the connections between operations and know which operation should be used in the given problems; and finally they can make mental computations and manipulate numbers in their head (Bresser & Holtzman, 1999).

2.2 The Addition and Subtraction

According to Piaget (1964), the essence of knowledge is an operation. To know a thing entails more than simply looking at it and mentally reproducing it. Furthermore, by knowing, he explained altering, transforming, and comprehending the object's transformation process, as well as the object's construction. The essence of knowledge is thus an operation, which is an interiorized activity that transforms the object of knowledge (Piaget, 1964). Since operations are the most fundamental components in elementary mathematics, they play a crucial role in following mathematics learning (Lee & Pang, 2012). The mathematical operation can be defined as a method of comparing numbers and influencing them on top of each other in accordance with a certain set of rules (TLI, n.d.).

There are four basic arithmetic operations namely; addition, subtraction, multiplication, and division. Specifically, the addition is described as the operation of combining numbers to obtain an equivalent simple quantity (Merriam-Webster, n.d.). The addition of whole numbers is the mental action of joining or combining two amounts, meanwhile, subtraction of whole numbers is the mental action of removing an amount from a larger amount (Sheffield & Cruikshank, 2005). Subtraction is defined as the process of taking one number or amount away from another number or amount (Cambridge University Press, n.d.). Just as addition is represented by a joining action, subtraction could be represented by a take away action (Battista, 1983).

Moreover, Kamii named the process of “making mental relationships between and among objects" as a constructive abstraction (1999, p. 16). Thus, according to the Kamii (1999) constructive abstraction could help students get a basic understanding of addition and subtraction.

2.2.1 The Problem Structures

Carpenter et al. (1983) stated that the most effective method to characterize students' problem solving processes was to link students' problem solving processes to the semantic structure of the problem. There are studies conducted by different researchers on problem structures. Although these researchers gave different names to the same problem structures, there are basically four different problem structures and Table 2.2. below shows these researchers, the semantic structures and corresponding definitions of these structures.

Table 2.2 The Problem Structures

The Semantic Structure	The Meaning of the Structure
Change	Increasing or decreasing the problem's original form to reach the final result (Carpenter et al., 1981, 1983, 1988; Greeno, 1980; Nesher, 1981; Nesher et al., 1982; Peterson et al., 1989; Vergnaud, 1982).
Joining & Separating	
Join & Separate	
Dynamic	
Transformation linking two measures	

Table 2.2 (continued)

The Semantic Structure	The Meaning of the Structure
Combine	Two different values that are parts of a whole are involved in a static relationship (Carpenter et al., 1981, 1983, 1988; Greeno, 1980; Nesher, 1981; Nesher et al., 1982; Peterson et al., 1989; Vergnaud, 1982).
Part-part-whole	
Static	
Composition of two measures	
Compare	The static comparison of two sets or one of the sets which the difference set is given (Carpenter et al., 1981, 1983, 1988; Greeno, 1980; Nesher, 1981; Nesher et al., 1982; Peterson et al., 1989; Vergnaud, 1982).
Comparison	
A static relationship linking two measures	
Equalize	Comparing the given sets, and to make one of the sets equal to the other, the problem asks how much it must be altered (Carpenter et al., 1981, 1983).
Equalizing	

As stated in Table 2.2. above, several different researchers have determined four categories of these semantic structures as change (join and separate), part part whole, compare and equalize. In the Change (join and separate) category, it has been emphasized that there must be an action which one set could join to another set or one set could separate from another set. Similar to Change category, Equalize category also needs some action to change one set to make it the same as another set. Conversely, Part Part Whole and Compare categories do not require an action. In other words, these are the problems that explain static relationships between quantities (Carpenter et al., 1981). Furthermore, the combination problems (part part whole) include relationships in a set, however comparison problems include comparing two different sets (Carpenter et al., 1988).

According to the types of relations involved, researchers have divided addition and subtraction problems into structures (Verschaffel et al., 2007). They also added that there are three different quantities in each problem which are the start, change, and result. Any of these quantities can be found if the other two quantities are given which means the unknown quantity may be the start, the change, or the result (Garcia et al., 2006).

Table 2.3. below shows the problem structures, different unknown quantities and related examples of those quantities.

Table 2.3 The Unknown Quantities in Problem Structures

	Problem Structures	Quantities	Examples
Addition and Subtraction	Join	Start Unknown	Sandra had some crayons. Richard gave her 4 more crayons. Now she has 12 crayons. How many crayons did Sandra have to start with?
		Change Unknown	Sandra has 4 crayons. How many more crayons does she need to have 12 crayons all together?
		Result Unknown	Sandra had 4 crayons. Richard gave her 8 more marbles. How many does Sandra have all together?
	Separation	Start Unknown	Sandra had some crayons. She gave 4 to Richard. Now she has 8 crayons left. How many crayons did Connie have to start with?
		Change Unknown	Sandra had 12 crayons. She gave some to Richard. Now she has 4 crayons. How many did Sandra give to Richard?
		Result Unknown	Sandra had 12 crayons. She gave 4 crayons to Richard. How many crayons does she have left?
	Part Part Whole	Part Unknown	Sandra has 12 crayons. Four are red and the rest are blue. How many blue crayons does Sandra have?
		Whole Unknown	Sandra has 4 red crayons and 8 blue crayons. How many crayons does she have?
	Compare	Difference Unknown	Sandra has 12 crayons. Richard has 4 crayons. How many more crayons does Sandra have than Richard?
		Large Unknown	Richard has 4 crayons. Sandra has 8 more than Richard. How many crayons does Sandra have?
		Small Unknown	Sandra has 12 crayons. She has 4 more crayons than Richard. How many crayons does Richard have?
	Equalize		Sandra has 7 crayons. If she buys 5 crayons, she will have the same number of crayons as Kelly. How many crayons does Kelly have?

Note. Adapted from “Teachers' Pedagogical Content Knowledge of Students' Problem Solving in Elementary Arithmetic” by Carpenter et al., 1988, *Journal for Research in Mathematics Education, 19(5)*, p. 388. Copyright 1988 by NCTM.

As seen in the Table 2.3. each structure has three quantities: a start, a change, and the result and these three numbers can be called as a “fact families”. For instance, 2,3,5 or 4,8,12. In a problem, either one of these three amounts could be unknown (Stocker Jr., 2021, p. 7). It has been stated that different combinations of the structures and the position of the unknown quantity has an impact on the cognitive processes of students to solve problems (Schoen et al., 2021). Furthermore, by using these problem structures several different problems could be created to ask student.

In regards to problem structures, this study does not include context base problem structures however it includes a variety of structurally-related problems. These problem types used in studies conducted by Peters et al. (2010) and Paliwal and Baroody (2020). Peters et al. (2010) described two different problem structures in accordance with the distances in a problem. In this case, Large Distance problems are where the difference between subtrahend and difference is bigger than ten, and Small Distance problems are where the difference between subtrahend and difference is less than ten. Besides, Paliwal and Baroody (2020) defined different part part whole related problems. Near Complement problems are the problems where the second addend is subtracted from the sum. For example, 3+8=11, 11-8=? is a near complement problem. Far Complement problems, on the other hand, are those in which the first addend is subtracted from the sum (e.g., 7+6=13, 13-7=?). Lastly, problems involving the same numbers in addition and subtraction but distinct part-whole relationships are described as decoy problems (e.g., 17 + 8 = 25, 17 – 8 =?). In their study, the researchers used fact families and developed part part whole related problems. Also, they focused on the complements of the numbers and inverse operations.

Students can utilize a known addition knowledge to determine an unknown subtraction fact or vice versa in a problem since addition and subtraction are related and complementary operations (Baroody, 1999). The teachers can teach the interconnectivity of the two operations by using fact families (Cobb, 1987). Fact families are number facts linked by two opposing operations like addition and subtraction and they help students in understanding how an addition fact may be utilized to find a subtraction fact or the other way round (Sun & Zhang, 2001). Therefore, the fact families are a strategy that can help students to improve in number and operations (Cobb, 1987; NCTM, 2000). Furthermore, it has been argued that fact families can navigate students to consider part-part-whole relationships while teaching them that subtraction and addition are complementary operations (Cobb, 1987; Zhou & Peverly, 2005).

2.2.2 Operation Sense

When it comes to children's subtraction, most of the developmental patterns that apply to children's addition also apply to children's subtraction (Hiebert et al., 1982). Hatfield et al. (2005) stated that an understanding of the meaning of operations could be developed along with knowledge of basic number combinations. In this matter, Slavit (1999) defined operation sense that comprises a set of flexible principles that the learner can link together. The underlying structure links with other mathematical operations and structures, and possible generalizations.

The following 10 components were proposed by Slavit (1999) as the overall concept of operation sense.

1. A conceptualization of the base components of the process.
2. Familiarity with properties which the operation is able to possess.
3. Relationships with other operations.
4. Facility with the various symbol systems associated with the operation.
5. Familiarity with operation contexts.
6. Familiarity with operation facts.
7. Ability to use the operation without concrete or situational referents.
8. Ability to use the operation on unknown or arbitrary inputs.
9. An ability to relate the use of the operation across different mathematical objects.
10. An ability to move back and forth between the above conceptions (pp. 254–258).

According to Slavit (1999) a flexible understanding of different components and properties of the operations is required for operation sense. With the ability of operation sense, students could be able to break down the operations into their fundamental meaning such as addition as counting or multiplication as repeated addition. They could comprehend the properties of operations (commutativity, associativity etc.) and invertibility between operations. Students with advanced operation sense could understand the addition, subtraction, multiplication and division facts so that they can build the connections among operations. In this matter, it has been argued that a lot of students are taught addition and subtraction facts before they can understand them (Hatfield et al., 2005). Furthermore, students can develop an operation sense by experiencing join, compare, and part-part-whole problems. When students are enacting their understanding of an operation on unknown quantities, they are displaying a higher level of operation sense (Slavit, 1999).

While students gain operation sense, they will develop a conceptual understanding where they build on the basic facts and finally algorithms. To produce invented strategies, the ability of knowing the operation facts is also considered as a necessity. As suggested in the PSSM (NCTM, 2000), exploring with numbers and operations helps children develop strategies to learn and recall basic facts.

2.2.3 Mathematical Proficiency in Operations

There are five strands of mathematical proficiency described as strategic competence, conceptual understanding, adaptive reasoning, procedural fluency and productive disposition (NRC, 2001). Students' efforts to manage their learning behaviors while they participate in classroom practices are defined as strategic competence. As a result, strategic competency entails understanding and applying strategies for analyzing and completing tasks and activities (Özdemir & Pape, 2012). The ability to comprehend mathematical concepts, operations, and relationships is known as conceptual understanding (NRC, 2001; Van de Walle et al., 2013). Conceptual understanding indicators include some concepts such as combining, decomposing and rearranging numbers, and making estimations (Van de Walle et al., 2013). Adaptive reasoning encompasses not just deductive reasoning but also intuition and inductive reasoning, which draws results based on patterns, similarities, and metaphors (NRC, 2001). Procedural fluency is the ability to perform procedures flexibly, accurately, efficiently and appropriately (NRC, 2001). The knowledge and application of mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately is referred to as procedural fluency (Hatfield et al., 2005). Using a range of solution strategies can sometimes be used to identify procedural flexibility or adaptivity (Verschaffel et al., 2007). The fifth and only affective disposition is productive, whereas the others are knowledge domain. It is described as the application of logic to rationalize a problem solution or to reach from something known to something unknown (Hatfield et al., 2005). The study's main point was that all five aspects of mathematical skills are interconnected and increase together (NRC, 2001).

The Table 2.4. below shows the name of the strands of mathematical proficiency, the technical terms and definitions of those strands.

Table 2.4 The Five Strands of Mathematical Proficiency

Name of Strand	**Technical Term**	**Definition**
Understanding	Conceptual Understanding	Comprehending mathematical concepts, operations, and relations. Knowing what mathematical symbols, diagrams, and procedures mean.
Computing	Procedural Fluency	Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
Applying	Strategic Competence	Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
Reasoning	Adaptive Reasoning	Using logic to explain and justify a solution to a problem or to extend from something known to yet known.
Engaging	Productive Disposition	Seeing mathematics as sensible, useful, and doable-if you work at it- and being willing to do the work.

Note. Adapted from *Mathematics methods for elementary and middle school teachers* (5th ed., p. 71) by Hatfield et al., 2005, Wiley. Copyright 2005 by NCTM.

As shown in Table 2.4. mathematically proficient students have both mathematical knowledge and the ability to use that knowledge in relevant situations (Schoenfeld, 2007).

Students who are mathematically proficient can:

-reason through with a problem on their own and find an entry point for the solution;

-rationalize variables in connection to the problem setting;

-develop arguments by using assumptions, definitions, and previous results;

-investigate all of the relevant problem solving strategies;

-speak clearly throughout discussions;

-find repetitive calculations and general approaches and shortcuts; and

-look for patterns (NCTM, 2014).

In the meantime, number sense is described as the ability to perform mathematical proficiency with numbers and quantities. An individual with number sense can express numerical ideas and solve problems in the number domain by using models, phrases, and diagrams to represent number concepts (Pope & Mangram, 2015). Similarly, Sowder (1992) was compiled a list of behaviors that occur in the presence of number sense based on the definitions and characteristics of number sense in the literature: combining and decomposing numbers, being flexible between different presentations, understanding the relative size of the numbers, using reference points, understanding the effects of operations on numbers, applying invented strategies while making the mental computation, estimation and making sense of numbers.

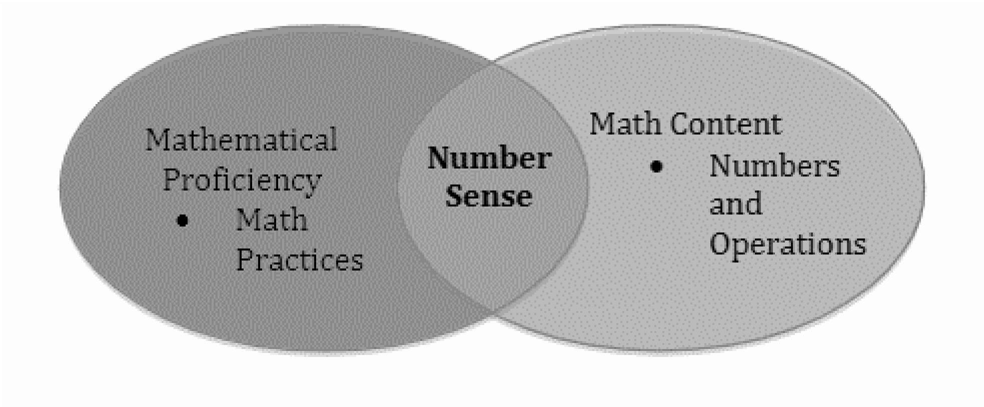


Figure 2.1 The relationship between mathematical proficiency and math content (Pope & Mangram, 2015, p. 7)

According to Figure 2.1 the common aspect of mathematical proficiency and mathematical content is the number sense. The ability to show mathematical proficiency with both quantities and operations is known as number sense (Pope & Mangram, 2015). It could be inferred that mathematical proficiency and number sense are connected and support each other.

2.3 Mental Computation

While the researchers could not agree on one definition of number sense, they did agree on the definition of mental computation skills. The procedure of doing arithmetical calculations without the use of external instruments is known as mental computation (Sowder, 1990). Particularly, mental calculations are conducted "in the mind" rather than "on paper," however this does not negate the need for symbolization to help mathematical reasoning (Harries & Spooner, 2000, p. 75). According to Yang and Huang (2014) mental computation skills are defined as the ability to solve everyday problems in a flexible and clear manner. It is a vital skill because it allows children to gain a better understanding of how numbers function, make judgments regarding procedures, and develop calculating strategies (Varol & Farran, 2007; Vershaffel et al., 2007). In addition, it has been also stated as an aspect of number sense (McIntosh & Sparrow, 2004).

The literature has suggested that adding mental computation in a mathematics curriculum that fosters number sense is critical (e.g., Maclellan, 2001; Reys et al., 1995). According to the NCTM (1989), paper and pencil algorithms might be given less attention, while mental computation, estimation, and the usage of calculators might be given more attention. Furthermore, Bums (1994) declared that enforcing the traditional arithmetic strategies on students is pedagogically dangerous because these strategies obstruct their learning process and encouraged them to assume that "mathematics is a collection of mysterious and often magical rules and procedures that must be memorized and practiced" (p. 472).

Some reformers argued that instead of teaching algorithms, some changes should be made and the individuals could be allowed to create their own procedures (Kamii et al., 1993). Carroll (2000) pointed out that there are two causes for this instructional change. One of these causes was that the students who might have built and explored their own meaningful operations are less likely to make the common errors that occur during the practice of some problems (Davis, 1990; Van Lehn, 1986). Another cause was that the process of inventing algorithms and discourse strengthens students' cognitive thinking and number sense (Sowder, 1992). Thus, the production of effective mental computation procedures can decrease or remove the requirement for formal written algorithms (McIntosh & Sparrow, 2004).

2.3.1 Mental Computation Strategies

In previous decades, researchers focused on how children develop certain concepts like counting, addition, and subtraction, and also how teachers might promote number learning. According to Markovitz and Sowder (1994), despite the crucial role of number sense in the development of mathematical thinking and life skills, mathematics curriculums focused on operational algorithms and processes. In addition to this, the findings of various research conducted in different countries confirmed this situation (Alsawaie, 2011; Kayhan-Altay, 2010; Reys et al., 1999; Yang et al., 2008).

In the literature, the number sense has been stated as it is related to one's ability to handle numbers, operations and everyday situations involving numbers. This skill is used to develop effective and flexible strategies for dealing with numerical problems (McIntosh et al., 1992). Instead of being taught how to execute written operations, students are advised to apply mental operations, investigate patterns, make predictions, and discuss possible connections when the number sense is included in mathematics instruction (Anghileri, 2006). In other words, students are encouraged to develop new ways by thinking creatively about the problem rather than relying on the usual algorithms and paper pencil techniques they have mastered. This enables them to come up with a variety of answers in the numerical circumstances they face on a regular basis (Şengül & Dede, 2014). The student's knowledge of some standard procedures for solving specific types of mathematical problems are still vital, but the emphasis is shifted to the individual's development of a diverse set of arithmetical principles and strategies and their effective and adaptive usage (Torbeyns & Verschaffel, 2016).

It's also vital to consider how students form, or fail to form, links between different strategies. Furthermore, this linking process needs students to uncover similarities and dissimilarities between one number strategy and another, and realizing these relationships will allow them to develop the flexibility that is necessary to establish a cohesive variety of computational strategies and the ability to switch between them (McIntosh & Sparrow, 2004).

Individuals with a developed number sense could produce different solutions by thinking flexibly without depending on traditional algorithms and written calculations, and this makes it easier for them to cope with numerical calculations they encounter in daily life (Çavuş-Erdem & Duran, 2015). The ability of problem-solving, knowing base-ten number principles, and flexibility in transmitting arithmetic knowledge may all make a contribution to a student's strategies (Clements et al., 2020). Number sense is important to students' subsequent math achievement (Olkun et al., 2019). Lower-achieving students mostly rely on counting procedures based on objects (eg. fingers or counters) or object representations (McIntosh & Sparrow, 2004). Whereas it can be expected that higher-achieving students have the tendency to use mental computation strategies.

The main goal of implementing reasoning strategies is to make students use the known facts and connections to answer unknown facts. There are two ways of doing this process; one is using a known fact (e.g., using 6+4 = 10 to solve an unknown fact 6+5= ?), the other one is using derived facts (e.g., when solving 6+5 by decomposing 6 into 5 + 1 and then finding 5+5+1) (Henry & Brown, 2008). The implementation of known or quickly derived number facts through combination with certain aspects of the number system to come up with a solution to a calculation whose result is unknown is what mental strategies are all about (Thompson, 1999).

Counting, estimating, grouping, partitioning, base ten and also other arithmetical procedures have all been explained in terms of fundamental parts of developing mental computation strategies (Olkun et al., 2019).

2.3.2 Addition and Subtraction Strategies up to 20

Students can develop several strategies while doing addition and subtraction up to 20 and these strategies can be varied from counting one by one to using some simple strategies that do not involve counting. Despite the fact that many low-achieving children fail to achieve the development, researchers recognize this development to simple addition and subtraction as essential mathematical learning (Ellemor-Collins & Wright, 2009).

Thompson (1999) described the addition and subtraction strategies up to 20 and the Table 2.5 below summarizes these strategies.

Table 2.5 The Addition and Subtraction Strategies up to 20

	The Type of Strategies	**The Strategies**	**Example**
Addition & Subtraction Strategies up to 20	Counting Strategies	Counting on from first	3+4=? 3,4,5,6,7 the answer is 7.
		Counting on from smaller	4+3=? 3,4,5,6,7 the answer is 7.
		Counting on from larger	3+4=? 4,5,6,7 the answer is 7.
		Counting back from	8-3=? 8,7,6,5 the answer is 5.
		Counting back to	8-3=? 8,7,6,5,4,3 the answer is 5.
		Counting up from (Complementary addition)	8-3=? 4,5,6,7,8 the answer is 5.
	Calculation Strategies	Double facts (subtraction)	14-7=? The answer is 7 because 7+7 is 14.
		Near doubles (addition)	9+5=? The answer is 14 because 9+9 is 18 and taking away 4, 14.
		Near doubles (subtraction)	9-4=? The answer is 5 because 10 taking away 4 is 6. 9 is one less than 10.
		Subtraction as the inverse of addition	8-3=? The answer is 5. Since 3+5 is 8.
		Using fives	6+8=? Taking 5 from 6, taking 5 from 8, and adding 1 and 3 to the 10. The answer is 14.
		Bridging through ten (addition)	7+5=? 7 is 3 less than 10. Subtract 3 from 5. The answer is 12.
		Bridging through ten (subtraction)	12-5=? Take 2 from 12 is 10. Take the left 3 from 10. The answer is 7.
		Compensation	9+6=? 10 and 6 is 16. 9 and 6 is 15. The answer is 15.
		Balancing	8+9=? Thinking 7+10 to find the answer. The answer is 17.

As seen in Table 2.5., the strategies can be investigated under counting strategies and calculation strategies. Counting strategies include counting on from the first number, counting on from larger, counting back from, counting back to and counting up from. Whereas, calculating strategies cover double facts, near doubles, subtraction as the inverse of addition, using fives, bridging through ten, compensation and balancing (Thompson, 1999).

2.3.3 Addition and Subtraction Strategies for Two Digit Numbers

When the studies about mental strategies are examined, it is observed that there are several similar strategies named differently by different researchers. Some of the researchers tried to classify these strategies under broad categories.

For example, according to Gülbağcı-Dede (2015) the strategies used while problem solving can be expressed under two categories and these are number sense strategies and rule based strategies. Similarly, The Common Core Standards (2012) differentiated these strategies under special strategies and general methods. These strategies can be explained as using numbers and operations flexibly in problems by using their own solution strategy without being bound by the rules and learned rules and standard written algorithms that do not require much reflection while solving problems, respectively.

In this regard, Olivier et al. (1990) have classified strategies in two categories. The first category was accumulative/iterative strategies. In this category compensation of the answer was done after one of the quantities in the problem were changed. For instance, 93-37 is answered by 90-30 and then added -7 and finally +3. The second category was replacement strategies. The alteration was applied into both quantities in the problem to get the same expression to solve the problem with less effort. For example, 93-37 is changed into 96-40.

Differently, Baroody (2006) conducted research on fundamental facts, outlined three stages in the learning process. The first stage was counting strategies by using physical objects or counting verbally to find the answer. The second stage was using logical reasoning to ascertain an unknown combination by employing known facts. Finally, the third stage was mastery in producing answers quickly and correctly. According to the researcher, in this stage, students could answer with "It is 11. I just know it." (p. 22). This stage was also called as fact retrieval strategy or retrieval strategy (Bush et al., 2015; De Smedt, 2016; Geary, 1999; Geary, 2003). The CCSS (2013) stated that students could know their facts from memory and this could be the outcome of using reasoning methods repeatedly rather than spending time on memorization.

Other classifications were made by different researchers, and they explained each strategy with examples in detail. The strategies of addition and subtraction that were stated from different researchers were given in Table 2.6 below.

Table 2.6 Addition and Subtraction Strategies for Two Digit Numbers

The Researchers	**The Strategy**	**Example**
Thompson (2000)	Partitioning	
Torbeyns et al. (2006), Selter (1998)	Split	
Beishuizen et al. (1997)	1010 (ten-ten)	52+45=? (50+40) + (2+5) = 97
Threlfall (2000)	Partial Sum	
Son (2016)	Partial Difference	57-21=? (50-20)+(7-1) = 36
Fuson et al. (1997), Torbeyns et al. (2009)	Decomposition	
Yang & Huang (2013)	Separation	
Thompson (2000)	Sequencing	
Torbeyns et al. (2006), Selter (1998)	Jump	
Threlfall (2000)	Cumulative Sum	45+33=? 45+30=75, 75+3 = 78
Beishuizen et al. (1997)	N10	
Son (2016)	Decomposing and taking away	44-27=? 44-20=24, 24-7 = 17
Fuson et al. (1997), Torbeyns et al. (2009)	Sequential	
Yang & Huang (2013)	Aggregation	

Tablo 2.6 (continued)

The Researchers	**The Strategy**	**Example**
Thompson (2000)	Hybrid	27+35=? 20+30+5+7=62 58-22=? 50-20+8-2=36
Beishuizen (1993), Torbeyns et al. (2006)	Split-jump	
Threlfall (2000)	Cumulo-partial Sum	
Beishuizen et al. (1997)	10S	
Thompson (2000)	Compensation	48+36=? 48+40=88, 88-4=84 52-15=? 55-15=40, 40-3=37
Son (2016)	Compensating	
Macintyre & Forrester (2003)	Overjump	
Fuson et al. (1997)	Overshoot and come back	
Beishuizen et al. (1997)	N10C	
Carroll (2000)	Change numbers in original problem	
Torbeyns et al. (2009)	Shortcut Compensation	
Yang & Huang (2013)	Holistic	
Thompson (2000)	Complementary addition	83-78=? 78+2=80, 80+3=83 the answer is 5.
Romberg (1992)	Shopkeeper arithmetic	
Son (2016)	Adding up	
Carroll (2000)	Add Up	
Beishuizen et al. (1997)	A10	
Torbeyns et al. (2009)	Shortcut Indirect Addition	
Paliwal & Baroody (2020)	Subtraction as Addition	
Carroll (2000), Fuson et al. (1997)	Standard Written Algorithm	48+26=? 8+6=14 40+20=60 60+10+4=74
Heirdsfield (2003)	Mental image of pen and paper algorithm	
Yang & Huang (2013)	Mental image of vertical addition subtraction	
Fuson et al. (1997)	Change Both Numbers	48+36=? 48+2, 36-2, 50+34=84
Threlfall (2000)	Bridging up through ten	56+37=? 37+3=40, 40 + 53 = 93

As stated earlier, various strategies were classified by the researchers, however, some of these strategies can be listed under the same meaning.

The partitioning strategy is known under different names such as split strategy (Torbeyns et al., 2006; Selter, 1998), partial difference (Son, 2016) or partial sum (Threlfall, 2000), decomposition (Fuson et al., 1997; Torbeyns et al., 2009), separation strategy (Yang & Huang, 2013) or 1010 as in the Deutch literature (Beishuizen et al., 1997). The 10s and other quantities in both numbers are partitioned by using the split strategy, and they are added or subtracted independently. This strategy is to add or subtract each separately and without regrouping the numbers (e.g., 39+15 = ?; 30+10 = 40, 9+5 = 14, 40+14=54).

The sequencing strategy is known under different names such as jump strategy (Torbeyns et al., 2006; Selter, 1998), decomposing and taking away (Son, 2016), sequential (Fuson et al., 1997; Torbeyns et al., 2009), cumulative sum (Threlfall, 2000), aggregation (Yang & Huang, 2013) or N10 as in the Deutch literature (Beishuizen et al., 1997). This strategy could be applied if one of the quantities in the problem is divided into smaller pieces as tens and units and gradually added to first tens then units or subtracted first tens and then units from the other quantity (e.g., 56+47 = ?; 56+40 = 96, 96+7 = 103).

The hybrid strategy is known under different names such as split-jump (Beishuizen, 1993; Torbeyns et al., 2006), cumulo-partial sum (Threlfall, 2000) or 10S as in the Deutch literature (Beishuizen et al., 1997). This strategy could be done by splitting both numbers into their component parts, adding the first set of components, and then gradually adding the remaining components (e.g., 56+37=?; 50+30=80, 80+6=86, 86+7=93).

The compensation strategy is known under different names such as shortcut compensation strategy (Torbeyns et al., 2009), compensating (Son, 2016), overjump (Macintyre & Forrester, 2003), overshoot and come back (Fuson et al., 1997;) change numbers in original problem (Carroll, 2000), holistic (Yang & Huang, 2013) or N10C as in the Deutch literature (Beishuizen et al., 1997). This strategy could be efficiently applied by either adding or subtracting a quantity that is larger than the stated number in the problem (56+29=?; 56+30=86, 86-1=85).

The complementary addition strategy is known under different names such as shortcut indirect addition (Torbeyns et al., 2009), subtraction as addition (Paliwal & Baroody, 2020), adding up (Son, 2016), add up (Carroll, 2000), shopkeeper arithmetic (Romberg, 1992) or A10 as in the Deutch literature (Beishuizen et al., 1997). This strategy could be applied by using complementary addition operations. In other words, by figuring out how much of the minuend should be added to the subtrahend (e.g., 64-58=?; 58+2=60, 60+4=64, 2+4=6).

The standard algorithm strategy is known under different names such as traditional algorithm (Bass, 2003), mental image of pen and paper algorithm (Heirdsfield, 2003) and mental image of vertical addition subtraction (Yang & Huang, 2013). Standard written algorithms use operations with digits rather than the actual magnitude of the quantities in the problem, which includes calculation of the difference between 6 and 4 instead of 60 and 40 when solving 68-43. It is well defined step by step processes for solving operations (Torbeyns & Verschaffel, 2016).

Some strategies do not fall into the above classifications. According to Fuson et al. (1997) there is a strategy called change both numbers. Understanding what needs to stay the same in each approach is necessary for this strategy. While making addition the total must be constant. To make one quantity simpler to add, this strategy just involves shifting some from one quantity to the other. Likewise, the exact quantity should be added to or subtracted from the difference in order to retain the difference in the subtraction operation (e.g., 78-56=?; 78+22=100, 56+22= 78, 100-78=22). Threlfall (2000) also introduced bridging up through ten strategies. In order to use this strategy, the cumulative sum must be directed at a ten or multiple of ten (e.g., 38+45=?; 38+2+43, 40+43=83).

There were some concerns stated in the literature in terms of using all of these strategies. Firstly, strategies should not be taught by a teacher, they should be acknowledged by students (Threlfall, 2000). In this regard, Carpenter et al. (1998) refer to the strategies that emerge during the mental calculation process as "invented strategy" (p. 4). Another concern is the procedure of making decisions, and it was emphasized the decisions should be made according to the problem characteristics. Although it was argued that there could be the best strategy for some problems, students could choose their strategy according to what is suitable for them, and they could switch between strategies and be flexible (Beishuizen, 1993).

2.4 Research on Number Sense and Mental Operation

Helping students develop their number sense in studies was acknowledged as a crucial task of mathematics education because of the significance of number sense (Anghileri, 2000; NRC, 1989; NCTM, 2000; Reys et al., 1999). Examining the relevant studies revealed that both students in primary school and teacher candidates have poor number sense on a national and international scale. The studies on this topic are listed as follows.

Reys and Yang (1998) investigated the relationships between arithmetic success and number sense of Taiwanese 6th and 8th grade students in their study. In a study conducted on 234 students, Taiwanese students showed very different levels of success in written calculations according to their number sense. The students were very successful in the calculations made with paper and pencil, but they could not achieve the same level of success when solving similar problems using the number sense approach.

On another study, Reys et al. (1999) studied for examining students' number sense in different countries such as Australia, Sweden, Taiwan, and the United Stated of America. They developed a number sense test and implemented to over 110 middle school students to each country. Even though country to country performances on the number sense problems varied, they detected poor performance of students across all nations consistently. The researchers found that mental computation with using reference points in answering questions is not a natural and easy method for many students.

Similarly, Alsawaie (2012) developed a study to investigate strategies adopted by academically successful students in United Arab Emirates. The data were taken from 30 academically successful 6 graders. 10 number sense problems that require basic arithmetic were given to the participants to solve and they were also interviewed. The findings revealed that only a small portion of solutions involved number sense skills like using benchmarks appropriately, utilizing numbers flexibly while mentally calculating, estimating, and making judgments the reasonableness of outcomes, understanding the effect of operations, and breaking down or rearranging numbers to find solutions. They discovered that students were heavily reliant on the rules they were taught in school.

Yang (2005) also encountered with this reliance on rule based approach. The researcher designed a study to evaluate the number sense of the Taiwanese six grade students. The selected students were low, middle and high achievement level students. It was determined that middle and low level achievement students tended to use rule-based methods. The findings showed that only a few number sense strategies were applied, regardless of performance level. Moreover, all of the students frequently used written standard algorithms and rule-based ways to explain their thinking. It was concluded that students' propensity for using paper and pencil methods constrained their capacity for thought and reasoning and this strong dependency on written algorithms seems to be a significant barrier to the growth of number sense.

In the doctoral study conducted by Kayhan-Altay (2010) in Turkiye, it was examined how the number sense of the 6th, 7th and 8th grade students changed according to the grade level, gender and number sense components. The study was conducted with 584 students; a test was designed which included the components of number sense and it was applied to the students. As a result of the study, it was revealed that the number sense of the students was quite low and the students generally preferred to do standard written algorithms and rule-based methods instead of number sense based approaches. In addition, a positive correlation was found between the mathematics performance of the students in the number sense test and their number sense scores.

Different from studies done with younger students some researchers focused on preservice teachers’ number sense and mental computation abilities. Yang et al. (2009) examined the strategies used by 280 Taiwanese primary teacher candidates in real life problem solving. As a result of the study, it was determined that one-fifth of the teacher candidates used number sense strategies such as using benchmark points and recognizing number sizes. In addition, it showed that the participants mostly preferred rule based strategies. The researchers stated that teacher candidates’ number sense is quite low and they emphasized that some precautions should be taken to increase their knowledge and use of number sense.

Besides, Yaman (2015) carried out a study to examine the number sense skills of 312 primary teacher candidates in terms of their levels at the university. Number sense test was used as a data collection tool in the research. The findings of the study revealed that participants number sense performances differed significantly according to different dimensions according to their grade levels. Mainly, the number sense performances of the junior and senior students were higher than the freshman and sophomore students. This situation was explained with the fact that Teaching Mathematics I and Teaching Mathematics II courses taught in the third year of the program.

Furthermore, Şengül (2013) also aimed to study the strategies used by the preservice primary teachers. To achieve this, the participants were asked to take the number sense test and the study was carried out with the participation of 133 preservice primary teachers. The test required mental computation in restricted given time and consisted five different components of number sense. The participants were also asked to explain their answers. The findings were analyzed using qualitative and quantitative analysis methods. As a result of this research, it was determined that the number sense of the preservice primary teachers was quite low. Furthermore, it was seen that the participants preferred rule-based strategies rather than number sense in every component of number sense.

There were some studies to investigate mastery on invented strategies of students and teacher candidates. Yang and Huang (2014) compared the mental calculation performances and mental strategies utilized by second graders before and after an educational intervention. The experimental group's students performed better on mental computations, according to the results. After the intervention, students in the experimental group were able to use more advanced mental strategies such as aggregation, holistic, short-jump, and unconventional strategies. According to these results, teaching the addition and subtraction with standard algorithm does not encourage the growth of mental computation.

On the other hand, Kabaran and Işık-Tertemiz (2019) studied to examine the number groups that primary school students in the second grade use when addressing flexible solutions for addition and subtraction as well as the methods they use. 56 primary school students took part in the study. According to the study's results, adding and subtracting with standard algorithm was the most common approach among students, while rounding to ten for addition and counting backwards for subtraction were the least common. It was revealed that the students were depended on the rule based strategies and standard algorithm.

Some studies reached the conclusion that the standard algorithm is the dominant mental strategy in comparison with other mental computation strategies; however, differently in these studies, the standard algorithm was used accurately and in a flexible way. For example, Carroll (2000) conducted one-on-one interviews with fourth graders and gave a test to assess their mastery of basic facts and multi digit calculations. They placed a focus on student invented strategies and discussions of possible solutions. Several problems that required the students to employ complex mental calculation approaches, like number breakdown or left-to-right addition. Moreover, most of the students employed the standard algorithm. Different from previous studies, the students' standard written algorithm and invented strategies were both remarkably precise.

Torbeyns and Verschaffel (2013) developed a study to analyze 21 Flemish students' mental computation strategy choices while doing addition and subtraction. The problems either encouraged the use of standard written algorithms or mental computation methods. The findings showed that students frequently used the standard algorithm for finding the results, even though the solution required using a mental computing methods. Additionally, they employed the standard algorithms with high efficiency and adaptability. This successful and flexible use of the standard algorithm was similar to the results of Carrol (2000)'s study.

In addition, mental computation strategies usage of students' and implementing the standard algorithm for multi-digit subtraction were examined by Torbeyns and Verschaffel (2016). 56 fourth graders were given a different set of subtractions that either encouraged the use of mental computation strategies or the standard algorithm.

Students could employ the standard algorithm or mental calculation to solve every problem whichever they preferred. Similar with studies developed by Carroll (2000) and Torbeyns and Verschaffel (2013), the results showed that even on subtractions designed to elicit mental computation, children of all performance levels utilized the standard algorithm frequently and accurately. Furthermore, accomplished students focused their strategy decisions on their unique mastery of the various strategies rather than adapting their choice of strategy to the characteristics of the problems.

Some researchers conducted studies by considering different characteristics of the problems and whether participants employed the related strategy or not. Torbeyns et al. (2008) studied how children who were traditionally taught use shortcut procedures with the numbers from twenty to one hundred. The study included 149 primary school students with different level of mathematical proficiency. They were given two assignments, each of which involved the indirect addition and compensation strategy. Firstly, students were told to use their chosen strategy to complete each item as quickly and precisely as possible. Secondly, they were asked to create at least two distinct strategies for each problem. The findings revealed that students of all academic levels and grades barely used the indirect addition and compensation strategy.

Similary, Peters et al. (2010) conducted a study to examine adults' use of addition to solve two-digit subtractions with the participation of 25 university students. They presented in the form of three variables (M-S=D) and these variables were minuend (M), to be determined difference (D) and the known subtrahend (S). They changed the relative magnitude of the subtrahend by presenting two-digit subtractions in both their matching addition format (9+_=77) and the conventional subtraction format (77-9=_). They have created two categories in terms of the numerical distance and they constituted these categories (S>D, S<D) for both addition and subtraction format. Also, they have separated the distances into large distance (difference between S and D > 10) and small distance (difference between S and D < 10). They also kept time while participant mentally calculate. They analyzed the results with regression models that predicted participants' reaction times based on various problem features. They contrasted results on two-digit subtractions shown in an addition format versus a subtraction format. There was no subtrahend-dependent choice between direct subtraction and subtraction by addition when the subtrahend and the difference were close to one another also they did not find a significant difference between addition and subtraction problems. However, they concluded large distance (e.g., 71-2=?) problems were answered more quickly and more accurately.

On the other hand, Paliwal and Baroody (2020) aimed to assess the effectiveness of structured instruction and shortcuts in fostering understanding and consistent application of the subtraction as addition strategy. To accomplish this goal, they developed a 12 week randomized control trial that included an experimental group that received structured subtraction instruction and practice, as well as two comparison groups that received unstructured practice of subtraction combinations and structured instruction and practice on a different type of reasoning strategy. They applied this study to 81 Grade K, Grade 1, Grade 2 and Grade 3 students. They categorized the problems as Near complement and Far complement. They also employed control trial decoys that comprised addition and subtraction tasks with the same numbers but different part and whole relationships, as well as addition and subtraction tasks that had no relation to one another. As a result, they claimed that near complement trials could use the subtraction as addition method. On the other hand, they noted that far complement trials required the application of subtraction as an addition approach. Moreover, since the shortcut does not apply to solving decoy problems, individuals were not expected to complete these tasks more quickly than others. The researchers reached the conclusion as complementary conceptual connection between addition and subtraction served as the foundation for merging these operations further into mental structure. Moreover, practice with one operation promoted mastery of the other.

Beishuizen et al. (1997) focused on two digit mental arithmetic up to 100 with differently presented problems. The study revealed that there are two frequently used calculation strategies. The first one is decomposing the tens and units in both numbers which was also known as 1010, and the other one is counting by tens up or down from the initial unsplit number also named as N10. Students in the third grade who consistently employ the 1010 or N10 strategies were given problems with indirect form 27 + _ = 65. These problems required strategy adjustment according to the characteristics of the problem. According to the findings, there are two distinct kinds of flexibility in strategy application and the first one is flexibility across strategies and the other one is flexibility within one strategy.

Güç and Karadeniz (2016) also carried out a study by using different characteristics of problems to find the strategies employed by primary school students when doing mental addition. 25 students participated in interviews for the study of the mental addition process. They found that instead of utilizing the correct method in line with the given numbers, it is seen that the students are applying the strategies they have accepted and are accustomed to. In the light of these results, considering the advantages of using different strategies in different situations, it was recommended to conduct studies on teaching the use of appropriate strategies according to the given problem.

Blöte et al. (2000) developed an experimental study to evaluate the flexibility in mental computation up to 100 with 60 participants who were 2nd graders. They made an intervention with realistic arithmetic education and also gave options of strategies to students. Different form previous studies, it was concluded that when the options are given to the participants, students' choice for particular mathematical strategies depended on the characteristics of the problems. Furthermore, the researchers claimed that this situation showed that students' conceptual comprehension of numbers was strong. However, the extent to which they really used these strategies was fairly limited. The majority of problems were handled with N10. For subtraction problems where the difference between two numbers was very small, a most of the students used short jump. Moreover, the participants solved addition problems less flexible than subtraction ones.

2.5 Summary of the Literature Review

This chapter presented the number sense concept, addition and subtraction operations, and mental computation concept. The number sense is the understanding of numbers and operations with the ability to apply that understanding in flexible ways and to evaluate and make mathematical judgements and invent efficient strategies for dealing with numbers and operations.

In a problem, there are three quantities and any of these quantities could be unknown. In the literature, the structures of addition and subtraction problems were stated and studies were made accordingly. The operation sense concept and mathematical proficiency are other competencies that show individuals' capabilities. Finally, the mental computation concept and the strategies were investigated thoroughly. The strategies were classified under addition and subtraction strategies up to 20, and strategies for two digit numbers. It has been emphasized that there is not a best strategy and the strategies should be invented by students what is appropriate for them. In addition, considering the studies in the literature, it is seen that the number sense levels of both students and preservice primary teachers are low not only in Turkiye but also in many parts of the world. Even though the designs of the studies are differentiated, most of the studies showed the dependency on rule based methods and standard algorithm in both students and preservice teachers. While some studies reached the accurate usage of the standard algorithm, most of the researchers concluded that this dependency on standard algorithm and rule based strategies indicates weak number sense and mental computation abilities and also inhibits the opportunity to improve these abilities.

CHAPTER 3

METHODOLOGY

The purpose of this research is to gain an in-depth understanding of preservice primary teachers' use of mental computation strategies on structurally-related addition and subtraction problems. To reach this purpose, the mental strategies used by preservice primary teachers while solving two-digit addition and subtraction problems that are related in terms of the part-part-whole structure was analyzed. In this chapter, the design of the study, the participants, the data collection tool, piloting data collection tool, the data collection procedure, the data analysis process, the role of the researcher, and the credibility and trustworthiness of the study will be addressed.

3.1 The Design of the Study

The main purpose of this study is to investigate mental strategies of preservice primary teachers. Specifically, the research questions aimed to understand which strategies they use when they compute mentally in part part whole related (e.g., 31-28=3, 3+28=?) two-digit addition and subtraction, and how the strategies change according to the different problem characteristics. To gain insight of their understanding and sense making qualitative study is conducted. This study was designed as a single case study with preservice primary teachers. A case study is a type of research approach where the researcher thoroughly examines a case or cases by using a variety of data sources to help explain a phenomenon (Creswell, 2009). Therefore, approaching the phenomenon from a number of aspects facilitates to understand it and reveals new elements of it (Baxter & Jack, 2008). Furthermore, the case was described by Miles and Huberman (1994) as phenomena that took place in a bounded setting and they added that the unit of analysis can be perceived as the case. The researchers exemplified that an individual/individuals, a role, a small group, an organization, a community or a nation could be a case.

According to Yin (2011), there are four different kinds of case studies. While the single-case holistic design and single-case embedded design examine only one case, the multiple-case holistic design and multiple-case embedded design examine more than one case. Additionally, holistic design includes only one unit of analysis on the other hand embedded design includes more than one analysis unit of the study. Consequently, in this current study, the case is preservice primary teachers' responses in structurally-related addition and subtraction problems. This study is embedded because the structurally-related addition and subtraction problems were analyzed under three categories: 1) Decoy, Far Complement, Near Complement, 2) Small Distance, Large Distance, 3) Subtraction, Addition. Hence, this study presents a single case embedded design Specifically, this study deals with which strategies the preservice primary teachers use while mentally calculate, how these participants employ these mental computation strategies and how they give meaning to the strategies. In short but in depth, this research investigates participants' way of thinking when making mental computation.

3.2 Participants

This research included 86 participants (63 females, 23 males) who are the undergraduate students at the Department of Primary Education one of the universities in Ankara. When the university placement success rankings in the last four years are examined, it is seen that the selected university is in the top three among primary education programs.

There were 345 preservice primary teachers in this primary education program. The participants were chosen by stratified random sampling. This sampling method aims to determine the subgroups of the target participants and ensure that they are represented in the selected participants. The total number of preservice teachers in the program, the proportions of the preservice primary teachers in the entire group of preservice teachers, and the number of selected participants were given in Table 3.1 below.

Table 3.1 The Number of Participants

	The number of PSTs in the program	**The proportions of PSTs in the program**	**The number of selected participants**
Freshman	83	.24	20
Sophomore	85	.25	21
Junior	84	.24	20
Senior	93	.27	25
Total	345	100	86

As seen in the Table 3.1, there were 83 freshmen teacher candidates (24% of the entire PSTs). The number of sophomores were 85 (25% of the entire PSTs). Similarly, junior teacher candidates were 84 (24% of the entire PSTs). Finally, senior teacher candidates were 93 (27% of the entire PSTs). When these proportions (24%, 25%, 24%, 27%) and the total number of preservice teachers (345) in the department considered, 20, 21, 20, 25 participants were selected respectively. As a result, 86 number of preservice primary teachers were chosen to be interviewed among 345 teacher candidates randomly. The classes were listed according to their student numbers. 20 preservice primary teachers were chosen randomly from year one students, 21 preservice primary teachers were chosen randomly from year two students, 20 preservice primary teachers were chosen randomly from year three students and finally 25 preservice primary teachers were chosen randomly from year four students. Out of 86 preservice teachers, 20 were freshmen (12 female, 8 male), 21 were sophomores (19 female, 2 male), 20 were juniors (12 female, 8 male), 25 were seniors (20 female, 5 male). Therefore, random variation was aimed to be established in data set (Miles & Huberman, 1994).

The participants were teacher candidates in the department of Primary Education. This department is a program that contains multiple disciplines. Primary teacher candidates not only get educated about teaching mathematics, teaching science, teaching Turkish language, and social sciences but also they learn teaching music, teaching art, teaching physical activities to students. The curriculum of the undergraduate students' program includes three lessons related with mathematics.

In the first semester of the program, Basic Mathematics in Primary Education course is given to the preservice teachers and the content of the course is "number systems, decimal, divisibility rules, the least common multiplier and the biggest common divisor, problems related to daily life requiring word problems, fractions and decimals, patterns, mathematical modelling, geometric shapes and objects, expansions of geometric objects, circumference , area and volume of geometric objects, basic units of measure". Basic Mathematics in Primary Education is worth 5 ECTS.

In the fifth semester of the program, Teaching Mathematics I is given to the preservice teachers. The aim and basic principles of mathematics teaching is "a brief history of mathematics teaching (in the world and in the Turkiye), major learning theories and their relationship with mathematics learning, strategies to be used in mathematics teaching, the scope, purpose and features of the primary school mathematics program, important skills in mathematics education, problem solving (strategies, stages, problem types, etc.), using information technologies, development of number concepts in children, place value, formation and structural properties of natural numbers, arithmetic operations, related subjects in primary school mathematics curriculum, objectives and applied studies to them". Teaching Mathematics I is worth 4 ECTS.

In the sixth semester of the program, Teaching Mathematics II is given to the preservice teachers. The course content is "fractions and its teaching (student difficulties in learning fractions, different meanings of fractions, fraction models, equivalence, comparison, ordering, operations with fractions, decimal fractions, operations with decimal fractions), geometry and its teaching, development of geometric thinking in children (2 and 3 dimensional geometry topics and their teaching), measurement and teaching (development of measurement idea in child, size, area, volume, time measurements, weighing, our money), data management, tables, graphs, measurement and evaluation in mathematics education (multiple measurement-evaluation methods and techniques), related subjects, objectives and related applications in the primary school mathematics curriculum". Teaching Mathematics II is worth 4 ECTS.

A primary teacher candidate graduates with a total of 240 ECTS. In this case, the place that mathematics teaching takes from the program is about 5.4%. Along with all of the course information given above, all of the participants have taken Basic Mathematics in Primary Education, 45 participants have taken the Teaching Mathematics I, and 25 preservice teachers have taken the Teaching Mathematics II course when this research took place. The researcher reached to randomly chosen 86 participants via WhatsApp and interviewed face to face.

3.3 The Data Collection Tool

This study gathered data by asking 48 problems to the participants. To decide those problems two different studies were combined. The first study was done by Paliwal and Baroody (2020) and it was aimed to assess the effectiveness of structured instruction in fostering understanding and consistent application of the subtraction as addition strategy. The researchers used different types of trials and sets in their study and these problems are given in the Figure 3.1 below.

Type of Trial	Trials	Set	Trial Number
Practice Trial	4 + 5 = 9, 5 + 4 = ?	A	1
	7 + 2 = 9, 2 + 7 = ?	B	1
Near-Complement Trials[a]	3 + 8 = 11, 11 – 8 = ?	A	2
	5 + 7 = 12, 12 – 7 = ?	B	7
	18 + 6 = 24, 24 – 6 = ?	A	7
	14 + 5 = 19, 19 – 5 = ?	B	3
Far-complement Trials[b]	6 + 4 = 10, 10 – 6 = ?	A	9
	7 + 6 = 13, 13 – 7 = ?	B	5
	4 + 15 = 19, 19 – 4 = ?	A	5
	5 + 16 = 21, 21 – 5 = ?	B	8
Decoy problems[c]	11 + 7 = 18, 11 – 7 = ?	A	6
	9 + 5 = 14, 9 – 5 = ?	B	2
	17 + 8 = 25, 17 – 8 = ?	A	3
	12 + 6 = 18, 12 – 6 = ?	B	9
Unrelated problems[d]	6 + 4 = 10, 8 – 3 = ?	A	4
	7 + 2 = 9, 9 – 4 = ?	B	4
	9 + 5 = 14, 8 – 5 = ?	A	8
	6 + 7 = 13, 11 – 6 = ?	B	6

Figure 3.1 The trials of Paliwal & Baroody (2020).

As seen in the Figure 3.1 Near Complement problems were the problems where the second addend was subtracted from the sum. For example, 3+8=11, 11-8=? was a near complement problem. On the other hand, Far Complement problems were the problems where the first addend was subtracted from the sum (e.g., 7+6=13, 13-7=?). Lastly, problems involving the same numbers in addition and subtraction but distinct part-whole relationships were described as decoy problems (e.g., 17 + 8 = 25, 17 – 8 = ?).

The second study was carried out by Peters et al. (2010) to examine adults' use of addition to solve two-digit subtractions. They presented in the form of three variables (M-S=D) and these variables were minuend (M), to be determined difference (D) and the known subtrahend (S). The Figure 3.2 below shows the problems that they asked the participants.

Numerical distance	S<D problems Subtraction format	S<D problems Addition format	S > D problems Subtraction format	S > D problems Addition format
Large distance	31 − 3 = .	3 + . = 31	31 − 28 = .	28 + . = 31
	34 − 8 = .	8 + . = 34	34 − 26 = .	26 + . = 34
	42 − 5 = .	5 + . = 42	42 − 37 = .	37 + . = 42
	52 − 4 = .	4 + . = 52	52 − 48 = .	48 + . = 52
	71 − 2 = .	2 + . = 71	71 − 69 = .	69 + . = 71
	77 − 9 = .	9 + . = 77	77 − 68 = .	68 + . = 77
	83 − 4 = .	4 + . = 83	83 − 79 = .	79 + . = 83
	93 − 5 = .	5 + . = 93	93 − 88 = .	88 + . = 93
Small distance	32 − 15 = .	15 + . = 32	32 − 17 = .	17 + . = 32
	43 − 18 = .	18 + . = 43	43 − 25 = .	25 + . = 43
	51 − 25 = .	25 + . = 51	51 − 26 = .	26 + . = 51
	53 − 24 = .	24 + . = 53	53 − 29 = .	29 + . = 53
	75 − 36 = .	36 + . = 75	75 − 39 = .	39 + . = 75
	81 − 37 = .	37 + . = 81	81 − 44 = .	44 + . = 81
	84 − 38 = .	38 + . = 84	84 − 46 = .	46 + . = 84
	92 − 44 = .	44 + . = 92	92 − 48 = .	48 + . = 92

Figure 3.2 The trials of Peters et al. (2010).

As shown in the Figure 3.2, the researchers changed the relative magnitude of the subtrahend by presenting two-digit subtractions in both their matching addition format (9+_=77) and the conventional subtraction format (77-9=_). They have created two categories in terms of the numerical distance and they constituted these categories (S>D, S<D) for both addition and subtraction format. Also, they separated the distances into large distance (difference between S and D > 10) and small distance (difference between S and D < 10).

For this current study the researcher initially considered Peters et al. (2010) problems and created new problems according to Paliwal and Baroody's (2020) categories and presented in the below tables (see Table 3.2 and Table 3.3). This set of problems constituted a pool for us to later select the problems that would be posed to primary preservice teachers.

Table 3.2 The Combination of the Studies S<D Problems

Numerical Distance	Peters et al. (2010) S<D problems: Subtraction format	Paliwal & Baroody (2020)			Addition Format	Paliwal & Baroody (2020)		
		Near Complement	Far Complement	Decoy Problems		Near Complement	Far Complement	Decoy Problems
Large Distance	**31 - 3 = 28**	**28 + 3 = ?**	28 – 31= ?	31 + 3 = ?	3 + 28 = 31	31 - 28 = ?	31 – 3 = ?	3 – 28 = ?
	34 - 8 = 26	**26 + 8 = ?**	26 - 34 = ?	34 + 8 = ?	**8 + 26 = 34**	34 - 26 = ?	**34 – 8 = ?**	8 – 26 = ?
	42 – 5 = 37	37 + 5 = ?	37 - 42 = ?	42 + 5 = ?	**5 + 37 = 42**	42 - 37 = ?	**42 – 5 = ?**	5 – 37 = ?
	52 – 4 = 48	48 + 4 = ?	**48 – 52 = ?**	52 + 4 = ?	**4 + 48 = 52**	52 - 48 = ?	52 – 4 = ?	**4 – 48 = ?**
	71 – 2 = 69	69 + 2 = ?	**69 - 71 = ?**	71 + 2 = ?	**2 + 69 = 71**	71 - 69 = ?	71 – 2 = ?	**2 – 69 = ?**
	77 – 9 = 68	68 + 9 = ?	68 – 77 = ?	**77 + 9 = ?**	**9 + 68 = 77**	**77 - 68 = ?**	77 – 9 = ?	9 – 68 = ?
	83 – 4 = 79	79 + 4 = ?	79 - 83 = ?	**83 + 4 = ?**	**4 + 79 = 83**	**83 - 79 = ?**	83 – 4 = ?	4 – 79 = ?
	93 – 5 = 88	88 + 5 = ?	88 – 93 = ?	93 + 5 = ?	5 + 88 = 93	93 - 88 = ?	93 – 5 = ?	5 – 88 = ?
		Paliwal & Baroody (2020)				**Paliwal & Baroody (2020)**		
		Near Complement	Far Complement	Decoy Problems		Near Complement	Far Complement	Decoy Problems
Small Distance	**32 – 15 = 17**	**17 + 15 = ?**	17 - 32 = ?	32 + 15 = ?	15 + 17 = 32	32 - 17 = ?	32 – 15 = ?	15 – 17 = ?
	43 – 18 = 25	**25 + 18 = ?**	25 - 43 = ?	43 + 18 = ?	**18 + 25 = 43**	43 - 25 = ?	**43 - 18 = ?**	18 - 25 = ?
	51 – 25 = 26	26 + 25 = ?	**26 - 51= ?**	51 + 25 = ?	**25 + 26 = 51**	51 - 26 = ?	**51 - 25 = ?**	25 - 26 = ?
	53 – 24 = 29	29 + 24 = ?	**29 – 53 = ?**	53 + 24 = ?	24 + 29 = 53	53 - 29= ?	53 - 24 = ?	24 - 29 = ?
	75 – 36 = 39	39 + 36 = ?	39 - 75 = ?	75 + 36 = ?	**36 + 39 = 75**	75 - 39 = ?	75 - 36 = ?	**36 - 39 = ?**
	81 – 37 = 44	44 + 37 = ?	44 - 81 = ?	81 + 37 = ?	**37 + 44 = 81**	81 - 44 = ?	81 - 37 = ?	**37 - 44 = ?**
	84 – 38 = 46	46 + 38 = ?	46 - 84 = ?	**84 + 38 = ?**	**38 + 46 = 84**	**84 - 46 = ?**	84 - 38 = ?	38 - 46 = ?
	92 – 44 = 48	48 + 44 = ?	48 - 92 = ?	**92 + 44 = ?**	**44 + 48 = 92**	**92 - 48 = ?**	92 - 44 = ?	44 - 48 = ?

*Questions in bold indicate the questions used in data collection

Table 3.3 The Combination of the Studies S>D Problems

Numerical Distance	**Peters et al. (2010)**							
	S>D problems							
	Subtraction format				**Addition Format**			
		Paliwal & Baroody (2020)				**Paliwal & Baroody (2020)**		
		Near Complement	Far Complement	Decoy Problems		Near Complement	Far Complement	Decoy Problems
Large Distance	**31 – 28 = 3**	**3 + 28 = ?**	3 - 31 = ?	31 + 28 = ?	**28 + 3 = 31**	31 - 3 = ?	**31 – 28 = ?**	28 – 3 = ?
	34 – 26 = 8	**8 + 26 = ?**	8 - 34 = ?	34 + 26 = ?	**26 + 8 = 34**	34 - 8 = ?	**34 – 26 = ?**	26 – 8 = ?
	42 – 37 = 5	5 + 37 = ?	**5 - 42 = ?**	42 + 37 = ?	37 + 5 = 42	42 - 5 = ?	42 – 37 = ?	37 – 5 = ?
	52 – 48 = 4	4 + 48 = ?	**4 - 52 = ?**	52 + 48 = ?	48 + 4 = 52	52 - 4 = ?	52 – 48 = ?	48 – 4 = ?
	71 – 69 = 2	2+ 69 = ?	2 - 71 = ?	71 + 69 = ?	**69 + 2 = 71**	71 - 2 = ?	71 – 69 = ?	**69 – 2 = ?**
	77 – 68 = 9	9 + 68 = ?	9 - 77 = ?	77 + 68 = ?	**68 + 9 = 77**	77 - 9 = ?	77 – 68 = ?	**68 – 9 = ?**
	83 – 79 = 4	4 + 79 = ?	4 - 83 = ?	**83 + 79 = ?**	**79 + 4 = 83**	**83 - 4 = ?**	83 – 79 = ?	79 – 4 = ?
	93 – 88 = 5	5 + 88 = ?	5 - 93 = ?	**93 + 88 = ?**	**88 + 5 = 93**	**93 - 5 = ?**	93 – 88 = ?	88 – 5 = ?
		Paliwal & Baroody (2020)				**Paliwal & Baroody (2020)**		
		Near Complement	Far Complement	Decoy Problems		Near Complement	Far Complement	Decoy Problems
Small Distance	**32 – 17 = 15**	**15 + 17 = ?**	15 - 32 = ?	32 + 17 = ?	17 + 15 = 32	32 - 15 = ?	32 – 17 = ?	17 – 15 = ?
	43 – 25 = 18	**18 + 25 = ?**	18 - 43 = ?	43 + 25 = ?	**25 + 18 = 43**	43 - 18 = ?	**43 – 25 = ?**	25 - 18 = ?
	51 – 26 = 25	25 + 26 = ?	**25 - 51 = ?**	51 + 26 = ?	**26 + 25 = 51**	51 - 25 = ?	**51 - 26 = ?**	26 - 25 = ?
	53 – 29 = 24	24 + 29 = ?	**24 - 53 = ?**	53 + 29 = ?	29 + 24 = 53	53 - 24 = ?	53 - 29 = ?	29 - 24 = ?
	75 – 39 = 36	36 + 39 = ?	36 - 75 = ?	75 + 39 = ?	**39 + 36 = 75**	75 - 36 = ?	75 - 39 = ?	**39 - 36 = ?**
	81 – 44 = 37	37 + 44 = ?	37 - 81 = ?	81 + 44 = ?	**44 + 37 = 81**	81 - 37 = ?	81 - 44 = ?	**44 - 37 = ?**
	84 – 46 = 38	38 + 46 = ?	38 - 84 = ?	**84 + 46 = ?**	**46 + 38 = 84**	**84 - 38 = ?**	84 - 46 = ?	46 - 38 = ?
	92 – 48 = 44	44 + 48 = ?	44 - 92 = ?	**92 + 48 = ?**	**48 + 44 = 92**	**92 - 44 = ?**	92 - 48 = ?	48 - 44 = ?

*Questions in bold indicate the questions used in data collection.

From the problems seen in the tables above, all of the problems were not included because sixty-eight problems were considered too much for an interview, too exhausting for the participants and could have effects on reliability of the answers. By consulting an expert, a total of forty-eight problems were selected, by choosing two from each category. With the expert’s opinion, the problems were chosen and the problems were considered as varied as possible.

When Paliwal and Baroody’s (2020) study was considered, there should be a shortcut problem so every card should contain problems in the form of one without unknown problem next to a result unknown problem. Moreover, this study included a variety of structurally-related problems, including decoy, far, near as category one, small distance, large distance as category two. Also, the first operations were given either in subtraction or in addition form according to the category three. Therefore, one problem had one of the characteristics under each category (e.g., 31-3=28, 28+3=? was a near complement, large distance, subtraction problem). These structurally-related problems were given in a card, and an example of a card (e.g., 31-28=3, 3+28=?) is given below in Figure 3.3.

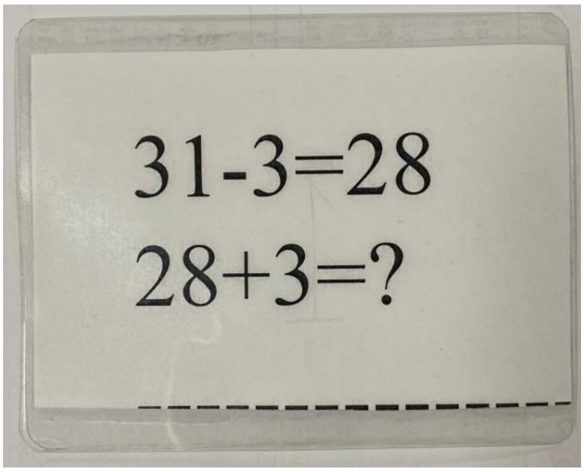


Figure 3.3 A data collection card example

The first card of the set is given in Figure 3.3. As seen in the figure the shortcut and a result unknown problem were given in the card. Thus, the list of problems was decided as whole forty-eight problem cards that contain shortcuts and result unknown problems. The sequence of the problems was arranged by consulting experts and the first twenty-four problems were decided as S<D, after twenty-four S>D problems were given. The final sequence of the problems was prepared by consulting experts and given below in the Table 3.4.

Table 3.4 The Data Collection Problems

	Card Number	Problem	Category 1	Category 2	Category 3
S<D	1	31-3=28, 28+3=?	Near	Large Distance	Subtraction
	2	34-8=26, 26+8=?			
	3	52-4=48, 48-52=?	Far		
	4	71-2=69, 69-71=?			
	5	77-9=68, 77+9=?	Decoy		
	6	83-4=79, 83+4=?			
	7	32-15=17, 17+15=?	Near	Small Distance	
	8	43-18=25, 25+18=?			
	9	51-25=26, 26-51=?	Far		
	10	53-24=29, 29-53=?			
	11	84-38=46, 84+38=?	Decoy		
	12	92-44=48, 92+44=?			
	13	8+26=34, 34-8=?	Far	Large Distance	Addition
	14	5+37=42, 42-5=?			
	15	4+48=52, 4-48=?	Decoy		
	16	2+69=71, 2-69=?			
	17	9+68=77, 77-68=?	Near		
	18	4+79=83, 83-79=?			
	19	18+25=43, 43-18=?	Far	Small Distance	
	20	25+26=51, 51-25=?			
	21	36+39=75, 36-39=?	Decoy		
	22	37+44=81, 37-44=?			
	23	38+46=84, 84-46=?	Near		
	24	44+48=92, 92-48=?			
S>D	25	31-28=3, 3+28=?	Near	Large Distance	Subtraction
	26	34-26=8, 8+26=?			
	27	42-37= 5, 5-42=?	Far		
	28	52-48=4, 4-52=?			
	29	83-79=4, 83+79=?	Decoy		
	30	93-88=5, 93+88=?			
	31	32-17=15, 15+17=?	Near	Small Distance	
	32	43-25=18, 18+25=?			
	33	51-26=25, 25-51=?	Far		
	34	53-29=24, 24-53=?			
	35	84-46=38, 84+46=?	Decoy		
	36	92-48=44, 92+48=?			
	37	28+3=31, 31-28=?	Far	Large Distance	Addition
	38	26+8=34, 34-26=?			
	39	69+2=71, 69-2=?	Decoy		
	40	68+9=77, 68-9=?			
	41	79+4=83, 83-4=?	Near		
	42	88+5=93, 93-5=?			
	43	25+18=43, 43-25=?	Far	Small Distance	
	44	26+25=51, 51-26=?			
	45	39+36=75, 39-36=?	Decoy		
	46	44+37=81, 44-37=?			
	47	46+38=84, 84-38=?	Near		
	48	48+44=92, 92-44=?			

After deciding the final form of the data collection problems, the limited time should be decided so it was consulted to the experts. They suggested that the time should not be too long since there are shortcuts for near and far complement problems but there should be enough time for decoy problems and for those who wouldn’t pay attention to the shortcuts. The experts suggested the time could be between 5 and 10 seconds for each card, which was tested in the pilot study. After the whole set was completed, it was decided to ask the participants how they tried to solve the questions that were answered incorrectly and for which given time was not enough.

3.3.1 The Piloting Data Collection Tool

Yin (2011) expressed that a pilot test is prior to the main implementation to clarify data collection plans and develop a relevant order of questions. Accordingly, in the present study data for the pilot study was collected in February 2021. The specific purposes of conducting the pilot study were to check whether forty-eight mathematics problems were appropriate for one-to-one clinical interviews, to estimate the necessary time for each mathematics problems, to decide on the implementation process, to be sure about the clarity of the statements before mathematics tasks.

The participants of the pilot study were selected through random sampling from a Primary Education undergraduate program of one of the state universities in Ankara. They were sophomores and juniors who had volunteered to participate in the study. To determine the necessary time, five seconds and seven seconds were given to each three participants. Initially, the plan was showing cards by the researcher but piloting data collection tool showed that the cards were not seen clearly by the participants. Since there was a pandemic and social distance should be taken into consideration, in the main data collection process the researcher decided to give the whole set of cards to the participant upside down so that the participants can answer questions one by one and wouldn't see the next question beforehand. To prevent skipping cards, the number of the problem should be given in the back of the card.

Before the piloting data collection tool, it was suggested by the experts the time could be between five and ten seconds for each card. It was planned to give five or seven seconds for each card which means after a card answered even within two seconds participants had to wait for the time out. In the pilot study, this process was seen as unnecessary. It has been observed out of six participants only one participant needed more than five seconds for some questions. However, overall the five seconds were appropriate for answering each card. Moreover, the researcher had to know all the answers so that she could separate wrong answers. In the pilot study, the researcher could separate the questions which time was insufficient for the participant but not the wrong answers. It was difficult to determine the right and wrong answers and keep track of the time, the assistance of someone was considered essential. Also, preparing an answer key table (see Appendix C) were considered as necessary checklist for the person who will help to differentiate the right and wrong answers.

While piloting data collection tool, it was seen that the clear instructions are vital. There were no examples before the forty-eight cards. Participants should be given problems to become familiar with the task administration. After seeing similar examples in the literature (Paliwal & Baroody, 2020; Peters et al., 2010) and understanding the importance of practice trials, two examples were chosen from the sets that were not used in the data collection cards to give in the main study beforehand. On the other hand, in the pilot study it was seen that some of the participants did not express any negative signs even when they found a negative answer and they explained that they did not think they had to say “minus 2” or “minus 4”. They just said the magnitude of the answer. In the main data collection process, the researcher emphasized that it is necessary to give the full answer. Furthermore, in the pilot study it was also seen that participants wasted some of their time with some expressions like “I am thinking like…” “I got excited, I can’t think” “Should I answer it now?”. This situation seemed problematic for the restricted time. It was planned that, before giving cards to the participants, the researcher should state this situation to the participants for not spending the time they had.

3.4 The Data Collection Procedure

Before carrying out the study, necessary permissions were obtained from Middle East Technical University's Human Subjects Ethics Committee (see Appendix A). The researcher also took the necessary permissions from the university where the data will be collected (see Appendix B). After the permissions, piloting data collection was conducted. The outcomes of the pilot study were taken into account when making the adjustments and procedures. The study was revised and restructured in light of the results from the pilot study. Then, the data was collected from February 2021 to April 2021. Participants were invited to one on one, focused clinical interviews. They were given volunteer participation form, they read it and signed it (see Appendix D). Besides, the researcher repeated the information given in the form verbally. They were explained the background of the researcher and the purpose of the study. It was reminded that they are volunteers of the study and the interviews will be audio recorded. The researcher emphasized that the interviews will only be evaluated by the researchers and their name will not be used anywhere. It was underlined that it was enough for the participants to say if they wanted to leave the study.

Before starting the interviews, the study was explained briefly and the necessary reminders were made. For instance, they were told they are expected to give full and clear answers in the given time. In addition, it was stated that it was enough for them to give only the answers first, and after the whole set was answered, they would be asked how they reached the wrong answers and answers that the time was not enough. The cards were given to the participant upside down and sorted from the first card to forty-eighth. There was an assistant present in the room and she marked the answers in the answer key checklist. The researcher kept time and separated the answers that were not given in five seconds. According to Sowder (1988), learned mathematics should be compatible with the claims, therefore Sowder (1988) suggested that the researchers should look at more than just the participants' responses and demand more explanations from them. Thus, after the whole problem set was finished, the researcher asked the participants how they found the answers of the problems that were answered incorrectly and were not answered in the given time. They were asked to elaborate their answers and way of thinking. Figure 3.4 below shows the data collection procedure.

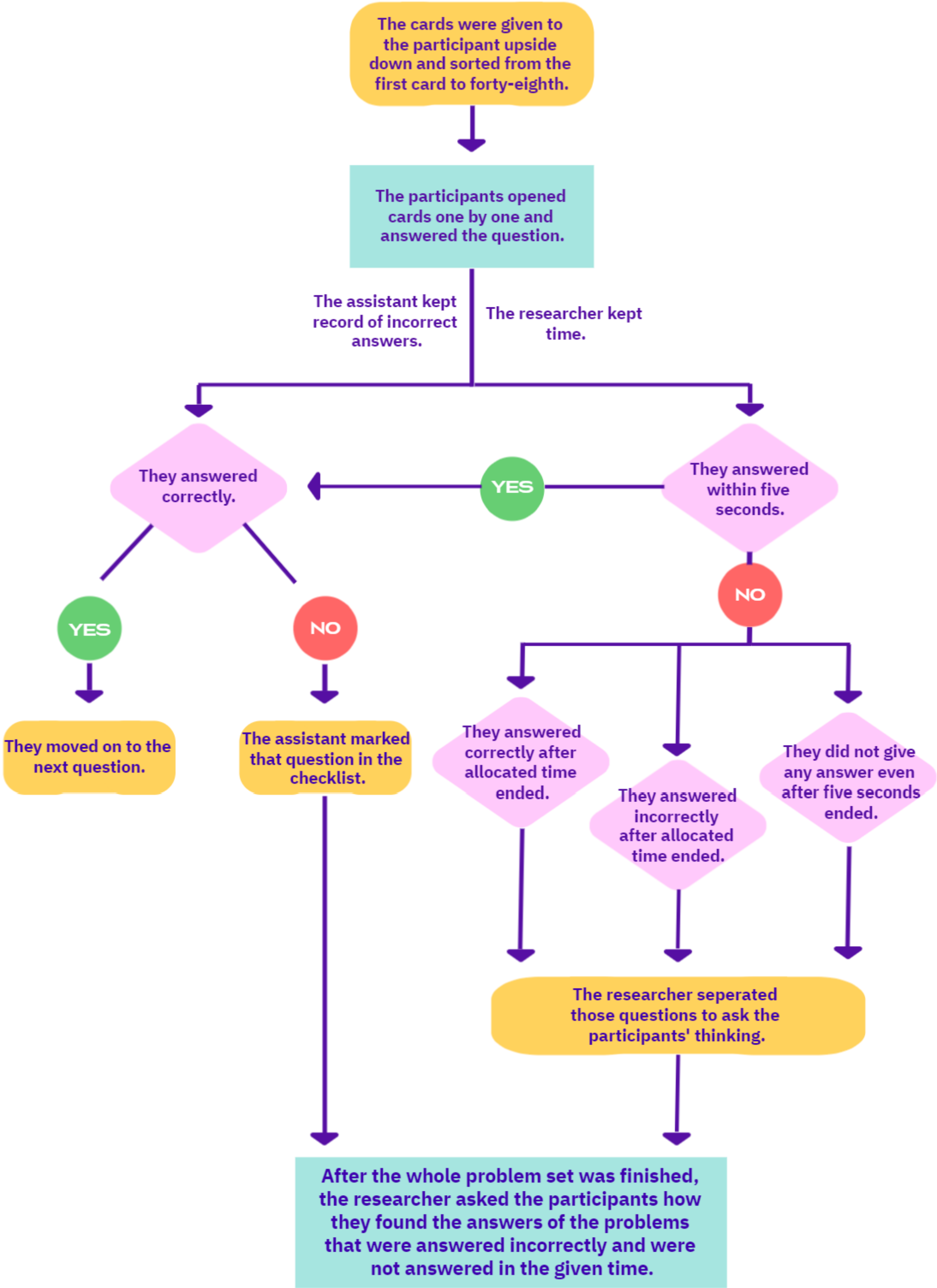


Figure 4.4 The data collection procedure

As seen in the Figure 3.4, preservice primary teachers' thinking processes and mental computation strategies on not manageable problems within and outside the allocated time were investigated in detail. In this process, the rules that were given in Figure 3.4. were followed. Moreover, participants were asked to detail their thinking and clinical interviews were conducted.

For example, for the 12th card (92-44=48 92+44=?), one of the clinical interview is given below.

> **Researcher:** How did you find this?
> **Participant:** How did I add it? I gave 8. That's 36. from 44. So I made it to 100.
> **Researcher:** You gave 8 to 92, you completed it to 100. OK. How do you do next?
> **Participant:** Then here I add what's left on 100.
> **Researcher:** Well, how many are left here?
> **Participant:** 36.
> **Researcher:** How do you know 36 left?
> **Participant:** I'm subtracting 8 from 44.
> **Researcher:** How do you subtract 8 from 44?
> **Participant:** Well.
> **Researcher:** How?
> **Participant:** I take it directly. I know it's easy because 44-8, 8 is a small number.
> **Researcher:** So you know it's 8 or do you trade? Or are you coming, for example, 4 and then 4?
> **Participant:** No. No, I know it by heart, and since I gave 8 here, I have to subtract 8 from it first and I subtract it directly. I know 44-8 is 36.
> **Researcher:** You know it's 36 when you get 8 out of 44.
> **Participant:** Ditto.
> **Researcher:** Okay, I got it. By giving 92 to 8, you rounded out to 100. You subtracted that 8 from 44 and added 100 and 36. Am I correct?
> **Participant:** Yes, exactly.

After their explanations, their strategies were summarized and they were asked whether the strategies were consistent with their thinking or not. All of the interviews were voice-recorded.

3.5 The Data Analysis Process

Approximately 18 hours (17 h, 25m, 37s) recordings were obtained. An interview lasted an average of 12 minutes. Initially, the audio recordings were transcribed and the assistant that was in the room also listened and controlled the transcripts. The data were analyzed with the descriptive analysis method. According to this approach, the data were summarized and interpreted according to predetermined themes and categories (Yıldırım & Şimsek, 2018). The transcripts were carefully examined numerous times from various angles while doing the descriptive analysis. As number of correct and incorrect answers (1) within the given period of time and (2) outside the given time were quantitatively (frequency, percentage based) analyzed, the mental computation strategies of preservice primary teachers were noted as codes. The mental strategies were handled in the Table 2.5. and Table 2.6. thoroughly. To name the mental strategies, the used strategies in the coding process is given below Table 3.5.

Table 3.5 The Mental Strategies Used in Coding Process

The Researchers	The Strategy	Example
Thompson (1999)	Counting on from first	3+4=? 3,4,5,6,7 the answer is 7.
	Counting on from larger	3+4=? 4,5,6,7 the answer is 7.
	Counting back from	8-3=? 8,7,6,5 the answer is 5.
	Counting back to	8-3=? 8,7,6,5,4,3 the answer is 5.
	Counting up from	8-3=? 4,5,6,7,8 the answer is 5.
Thompson (1999)	Double facts (subtraction)	14-7=?, the answer is 7 because 7+7 is 14.
	Near doubles (addition)	9+5=?, the answer is 14 because 9+9 is 18 and taking away 4, 14.
	Near doubles (subtraction)	9-4=?, the answer is 5 because 10 taking away 4 is 6. 9 is one less than 10.
Thompson (1999)	Using fives	6+8=?, taking 5 from 6, taking 5 from 8, and adding 1 and 3 to the 10. The answer is 14.
	Bridging through ten (addition)	7+5=?, 7 is 3 less than 10. Subtract 3 from 5. The answer is 12.
	Bridging through ten (subtraction)	12-5=?, take 2 from 12 is 10. Take the left 3 from 10. The answer is 7.
Thompson (2000)	Partitioning	52+45=? (50+40) + (2+5) = 97 57-21=? (50-20)+(7-1) = 36

Table 3.5 (continued)

The Researchers	The Strategy	Example
Thompson (2000)	Sequencing	45+33=? 45+30=75, 75+3 = 78 44-27=? 44-20=24, 24-7 = 17
Thompson (2000)	Hybrid	27+35=? 20+30+5+7=62 58-22=? 50-20+8-2=36
Thompson (2000)	Compensation	48+36=? 48+40=88, 88-4=84 52-15=? 55-15=40, 40-3=37
Paliwal & Baroody (2020)	Subtraction as Addition	83-78=?, 78+2=80, 80+3=83 the answer is 5.
Carroll (2000), Fuson et al. (1997)	Standard Written Algorithm	48+26=? 8+6=14 40+20=60 60+10+4=74
Fuson et al. (1997)	Change Both Numbers	48+36=? 48+2, 36-2, 50+34=84
Carpenter et al. (1981), Peterson et al. (1989)	Part Part Whole	Two different values that are parts of a whole are involved in a static relationship.
Baroody (2006), Geary (1999)	Fact Retrieval	Providing responses efficiently and accurately by saying, "I just know it."

As given in the Table 3.5., the strategies were mostly used based on Thompson's (1999, 2000) framework. In addition to the Thompson (1999, 2000), studies of Paliwal and Baroody (2020), Carroll (2000) and Fuson et al. (1997) were used. Moreover, Thompson's (1999) Using fives and Bridging through ten strategies named under Benchmark strategy. Also, the answers, which were clearly expressed as a whole and in two parts, were classified as Part Part Whole strategy and participants' answers where they stated their strategy with "I just know it." evaluated as Fact Retrieval. In addition to these strategies, some of the strategies emerged from preservice teachers' answers in this study and these strategies were coded as well. These emergent strategies were named and described as follows:

- ***Adding Tens Strategy*** can be described as adding the ones first, then adding addend ten by ten. For example, while solving 92-44=48, 92+44=? PST1.18 explained their reasoning as "I added 4 to 92, then it makes 96. I add another 10 to pass 100. 106, 116, 126, 136.". The ones who used this strategy just focused on adding ten by ten and they did not try to make the first addend tens; even if they passed through tens, they continued to add ten by ten.

- ***Skip Counting*** observed in 22nd question (37+44=81, 37-44=?) exactly four times. For example, PST2.10 explained their reasoning “These are multiples of 7 directly, I found it from here. If 37, then 44. Like 7,14.” and another participant PST2.11 stated their thinking as “Now I'm going from multiples of 7, saying 7,14,21.”.
- ***Making Similar*** can be described as making the ones same in order to reach answer that ends with tens in subtraction. For example, when solving 25+18=43, 43-25=? PST1.20 stated that “Here, I can write 25 as 43-23-2. Then it will be 20-2 18.” or PST2.4 elaborated their answer to the 38+46=84, 84-46=? problem by saying “I thought like this. Subtract 44 from 84 and you get 40. I'm subtracting 2 more from 40.”. This strategy is like Compensation strategy but in a more sophisticated way.
- ***Reverse Sequencing*** is the inverse of Sequencing strategy. Sequencing is adding/subtracting the tens first and then adding/subtracting the ones. On the other hand, Reverse Sequencing is adding/subtracting the ones and then adding/subtracting the tens. For instance, while solving 84-38=46, 84+38=? PST2.11 explained their thinking as “First I add 84 to 8, then add 30.”. or PST3.14 stated the solution process of 25+18=43, 43-25=? problem as “I'll subtract 5 so 38. Then I'll subtract 20 as well.”.
- ***Switching*** is another preservice teachers’ unexpected strategy. This strategy is like the Change Both Numbers strategy but in a more specific way. It is changing the places of ones between the first addend/minuend and the second addend/subtrahend. For example, while answering 77+9 problem, finding 79+7. PST3.11 and PST4.10 answered the same problem with this strategy. They detailed their understanding as “First I think of 77 as 79, not 77. I replace 7 with 9. I will complete the remaining 7 later. 79, 80, 86.” and “I think like 79+7 because it seems easier to count 7 on top of 9 than counting 9 on top of 7.” respectively.

- ***Thinking Symmetrical*** came out in large distance (where D-Sb>10 in M-Sb=D) problems (e.g., [52-4=48, 48-52=?], [71-2=69, 69-71=?] and [4+79=83, 83-79=?]). For instance, PST2.7 explained their reasoning as "It says 2 minus 2 plus. Because it's a symmetrical thing, ma'am. This comes out automatically." or PST4.17 expressed "There are 2 out there to round 48 to 50. 2 is here as well 4.". While using this strategy, the preservice teachers think of the exactly number in the middle of minuend and subtrahend and they operate on that number symmetrically.
- ***Standard Algorithm but Different Order Strategy*** is one of the observed strategies used by the preservice teachers in the problems they answered wrong and did not have enough time. Unlike the standard algorithm, preservice teachers compute tens first, then move on to the ones. For example, PST3.9 described their strategy for solving 18+25=43, 43-18=? "I subtracted the tens first. That's how I usually do it. I subtracted 10 from 40. 30 left. I put a ten next to the 3. 8 out of 13, 5 left. I'm dropping a ten. The answer is 25 here." or PST4.15 explained their solution process for the same problem as "I start with tens, but with 1 less. Because 8 will subtract from 3, so I start by subtracting 1 from tens. I say 3, I subtract 1 from 3, 2, then 8 subtract from 13 is 5. Sometimes this happens involuntarily."

The codes were the strategies and those were drawn from participants' verbal explanations. The codes and strategies were controlled and reviewed by an expert. The categories were already regulated as Category 1 (Near Complement, Far Complement, Decoy), Category 2 (Large Distance, Small Distance), and Category 3 (Subtraction, Addition) and analysis were made accordingly.

While making analysis, the preservice primary teachers were named according to the year in the program and the order of the participation. For example, PST1.1 is a year one preservice teacher and 1st participant of the study or PST2.4 is year two preservice teacher and 4th participant.

3.6 Researcher Role

Johnson (1997) stated that researcher bias can have a potential threat to validity in qualitative studies and it can affect the results of the research. Additionally, emphasizing the importance of the researcher's involvement in qualitative research, Creswell (2009) advised that a researcher should be open and honest about his or her background, prior experiences, and interactions with participants. Therefore, the researcher explained what the study is about, what is the purpose of the study, where the data will be used and how it will be handled, also the researcher give detailed information about her background and research interest. The researcher also stated to the participants that the voice recordings will not be shared and their names will be anonymous in this study.

The researcher conducted the clinical interviews with the preservice primary teachers. For this method it is significant to recognize the value of clarification while posing questions. Participants can discuss their mathematics and give explanations for their behaviors during clinical interviews (Hunting, 1997). During the first period of the interview which was answering problems, the researcher kept time and separated the problems that the time was not enough. Since there was another person present in the interview to keep track of the right and wrong answers, it was helpful to control the researcher bias.

Although there were forty-eight problems to ask the participants, while they were explaining their solution ways for incorrectly answered problems, the researcher asked some additional questions to elaborate on their solution processes and thinking. Sometimes researcher also needed to ask the meaning of the used term by asking “What do you mean by saying …?”. Occasionally, some of the statements of their way of thinking was not clearly explained, in those times the researcher asked to repeat the explanation and define explicitly. The researcher especially paid attention to not to express any opinion about the answers, and tried to remain neutral during the interview to reduce the bias. Even when they answered and waited for the researcher’s approval, the researcher did not give any reaction to not to affect the study.

3.7 Credibility and Trustworthiness

The four types of validity and reliability problems of a qualitative study that the researchers identified were listed below (Lincoln & Guba, 1985; Miles & Huberman, 1994; Yıldırım & Şimşek, 2018). These four types were briefly covered because the study's design was qualitative.

In the validity concepts, there are internal validity and external validity. The first concern is internal validity and it requires credibility. In the present study, deep focused interviews, triangulation, expert opinions, member checking were used to ensure the internal validity of the study. The interviews should be done with deep and focused perspective and to ensure credibility clinical interviews were implemented. There are three types of triangulation methods to provide credibility. The first one is multiple data types. To ensure this, researcher gathered audio recordings, researcher notes and checklist. The second triangulation is multiple researchers and to provide this researcher consulted experts not only in the process of developing data collection tool but also in the process of data analysis. The third triangulation is multiple theory/perspective. To develop codes of the mental strategies, the framework of multiple researchers was applied. Finally, to provide credibility, member checking was applied and the researcher summarized the data she has collected and ask the participants to express their thoughts on the accuracy of them. Thus, credibility of this study was provided. The second concern is external validity and it requires transferability. To ensure this, detailed description of the participants' thinking and direct quotations were given.

In the reliability concepts, the first concern is dependability. To ensure this, experts contributed to the study and reviewed the data. Moreover, the analysis was made by examining the data several times repeatedly and direct quotations were included in the findings. The second concern is confirmability. The study was described thoroughly and direct quotations of the participants were given. Furthermore, there were another person to keep the answers in the room along with the participant and the researcher. That person was also reviewed the transcripts of the data.

CHAPTER 4

FINDINGS

The aim of this study is to determine the strategies used by preservice primary teachers mentally solving two-digit addition and subtraction problems that are related in terms of part-part-whole structure. To accomplish this purpose, preservice primary teachers were asked 48 problems in a given time. They were asked to elaborate the strategies they used in the problems they answered incorrectly and the strategies they used in the problems where the given time was not enough. In this chapter, the findings of the qualitative data collected from preservice primary teachers are presented. The total participants were 86 college students. This chapter contains a) performances of the preservice primary teachers b) the strategies that were used by the participants c) the strategies across problem categories.

4.1 The Performances of the Preservice Primary Teachers

The findings of the first research question which was “What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems?” addressed in the sections below. First of all, the performances within the allocated time were examined, then the performances outside the allocated time were investigated. Each level of preservice primary teacher performances from freshman to senior levels were presented in detail. Also, the overall performances of preservice primary teachers in solving structurally-related two-digit addition and subtraction problems were examined thoroughly.

4.1.1 The Performances of the Preservice Primary Teachers Within Allocated Time

The preservice primary teachers' answers were analyzed. The participants were asked 48 questions in total and 86 preservice primary teachers were the participants. Table 4.1. shows the average performances of preservice primary teachers in solving structurally-related two-digit addition and subtraction problems.

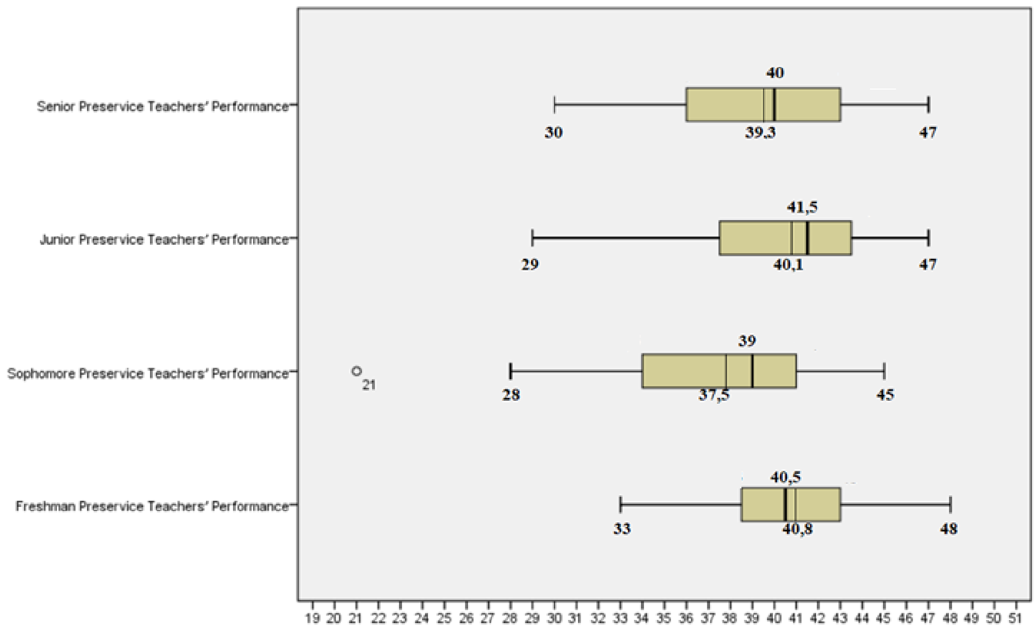
Table 4.1 Performances of the Preservice Primary Teachers

PSTs	Performances	Number of correct answers		Number of incorrect answers		Number of unanswered questions
		Within the given time period	Outside the given time period	Within the given time period	Outside the given time period	
Freshman PSTs	**Average**	40.8 (85%)	1.5 (3.1%)	3.4 (7.1%)	0.7 (1.6%)	1.6 (3.2%)
Sophomore PSTs	**Average**	37.5 (78.1%)	1.6 (3.4%)	5.3 (11%)	0.6 (1.3%)	3 (6.2%)
Junior PSTs	**Average**	40.1 (83.5%)	1.4 (2.8%)	3.9 (8.1%)	0.9 (1.9%)	1.7 (3.7%)
Senior PSTs	**Average**	39.3 (81.8%)	2.1 (4.4%)	4.4 (9.1%)	1.4 (2.9%)	0.8 (1.8%)

*Out of 48 questions

When the performances were analyzed, it was seen that the average score was the highest in freshman preservice teachers since they answered 85% of the questions correctly and within allocated time. Moreover, they had 40.8 average score out of 48 questions. On the other hand, sophomore preservice primary teachers were the least successful comparing with other levels. They answered 78.1% of the problems accurately and within the given time period. They successfully answered 37.5 out of 48 problems average. Junior and senior preservice primary teachers' performances were close but juniors had slightly more success than the seniors. While junior preservice primary teachers answered 83.5% of the problems correctly and within the restricted time and senior preservice primary teachers answered 81.8% of the problems. On the other hand, it was observed that some problems were answered within the given time period but incorrectly. These scores were similar with the correct answers within allocated time. In this case, sophomore preservice teachers answered 11% of the problems within the allocated time but incorrectly. Furthermore, this was the highest score among participants. Otherwise, freshman preservice teachers answered only 7.1% of the problems incorrectly and within the restricted time. This score was the lowest score in comparison with the other preservice teachers. In terms of average, while sophomore preservice teachers answered 5.3 out of 48 questions incorrectly and within the limited time, this score was 3.4 out of 48 questions in freshman preservice teachers. Additionally, as given in Table 4.1 there were some questions that were answered outside the allocated time. These questions were investigated in the following sections of this chapter.

The performances of preservice teachers were also summarized visually based on quantitative descriptions. The Figure 4.1 below shows the performances of the participants in box plot.



Note. Thick lines indicate median values, thin lines indicate mean values

Figure 4.1 The performances of preservice primary teachers on box plot

According to Figure 4.1, minimum and maximum values in every level can be understood. For example, while freshman participants had the highest maximum value, sophomores had the least maximum value among preservice teachers. In terms of minimum values, it can be seen that sophomores had the least minimum value, which was also an outlier; on the other hand, freshmen had the greatest minimum value. Moreover, we can also observe the median values from Figure 4.1. Freshman, junior and senior preservice teachers' performances median values were close, however sophomores had lowest median value in comparison with other levels. Also, all the median values except freshman preservice teachers' were greater than their mean values. Furthermore, the overall range of the data set was large for all levels except seniors based on the distances between the ends of the two whiskers for each box plot and this result was similar in terms of the interquartile ranges according to the box lengths.

The preservice primary teachers' answers were also examined according to the categories. Table 4.2. was developed by taking into consideration only the correct answers within the allocated time.

Table 4.2 Preservice Teachers' Number of Correct Answers in Particular Categories

PSTs	**Performances**	Category 1*			Category 2*		Category 3*	
		Decoy	Far	Near	SD	LD	Subtraction	Addition
Freshman PSTs	**Average**	13.6/16	13.4/16	13.7/16	19.5/24	21.2/24	20.1/24	20.6/24
	%	85.3	83.7	85.9	81.4	88.5	83.9	86
Sophomore PSTs	**Average**	12.9/16	12.1/16	12.5/16	17.6/24	19.9/24	19.2/24	18.2/24
	%	80.4	75.6	78	73.2	83	80.2	76
Junior PSTs	**Average**	12.2/16	13.3/16	14.5/16	19.1/24	20.9/24	19.7/24	20.3/24
	%	76.5	83.4	90.6	79.8	87.2	82.2	84.8
Senior PSTs	**Average**	12.5/16	13.3/16	13.4/16	19.1/24	20.2/24	19.7/24	19.6/24
	%	78.2	83.2	84	78.7	84	82.1	81.5

According to Table 4.2., when the category one was considered, preservice teachers' most of the correct answers differentiated among different years. Freshman preservice participants' most of the correct answers fell into the near category and the least number of correct answers was in the far complement category. On the other hand, junior and senior participants' correct answers were mostly in near complement and the least in decoy problems. Differently, sophomore preservice primary teachers' most of the correct answers fell into the decoy problems and the least number of correct answers was in far complement. The problems with the most number of correct answers in category two were large distance problems in every group of participants. Finally, considering category three, the scores were close between subtraction and addition types. However, the problems with the most number of correct answers were subtraction operation in sophomores and seniors, and least number of correct answers were addition problems in freshmen and juniors.

The findings related to the first research question were also examined by considering all grade levels together as overall scores. Table 4.3 summarizes the overall performances of preservice primary teachers in solving structurally-related two-digit addition and subtraction problems.

Table 4.3 The Overall Performances of the Preservice Primary Teachers

	Number of correct answers		Total number of correct answers	Number of incorrect answers		Number of unanswered problems	Total number of incorrect and unanswered problems
	Within the given time period	Outside the given time period		Within the given time period	Outside the given time period		
Average	39.4	1.7	41	4.2	0.9	1.7	6.9
%	**82**	**3.5**	**85.5**	**8.9**	**1.9**	**3.7**	**14.5**

*Out of 48 questions

According to the Table 4.3., the total number of correct answers was 41 on average and 85.5% of the problems were answered correctly within and outside the allocated time. 82% of this score was within the given period of time and 3.5% of it was outside the allocated time. The total number of incorrect and unanswered problems was 6.9 on average with 14.5% percentage score, of which 8.9% were within the given time period, 1.9% were outside the given time period, and 3.7% were unanswered. As a conclusion, the preservice primary teachers showed 82% success by giving correct answers in the given time period.

Overall scores were also investigated according to the different characteristics of the problems. Figure 4.2 given below shows the overall number of correct answers in particular categories.

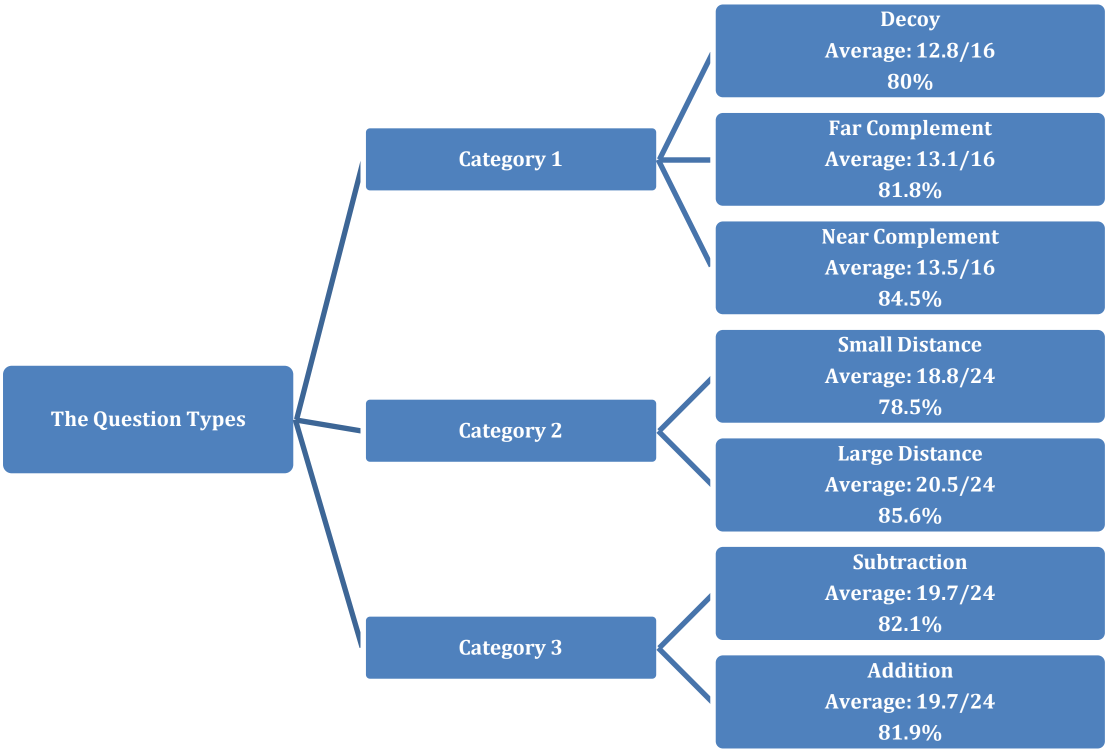


Figure 4.2 Preservice primary teachers' overall number of correct answers in particular categories

There were 86 participants in this study, and they were asked 48 problems in total. In category one, there were 16 problems under each type. On the other hand, in category two and category three, each type had 24 problems. According to Figure 4.2, when category one was considered, preservice teachers' most of the correct answers fell into the near complement category (84.5%) and the least number of correct answers was in the decoy category (80%). The problems with the highest number of correct answers in category two were large distance problems (85.6%) and the problems with the lowest number of correct answers were small distance problems (78.5%). Finally, considering category three, even if the total scores were close to each other, the problems with the most number of correct answers were subtraction operation problems (82.1%) and the least number of correct answers were addition operation problems (81.9).

To conclude this section, when the performances of the preservice primary teachers were summarized, it was observed that 82% of the preservice primary teachers' answers were correct and given within time in the overall results. Considering the grade levels separately, freshmen preservice teachers showed the highest success by answering 85% of the questions correctly within the given time. On the other hand, sophomores had the least successful performance with a 78.1% success rate. In overall findings, near questions were answered most accurately, this situation was also present in every grade level except sophomores. Sophomores answered decoy questions most precisely. Interestingly, decoy problems were the least successful type among other categories in junior, senior and overall results. When it comes to the distance type, large distance problems were answered more successfully than small distance problems in each grade level. Furthermore, it was also revealed that the performances were very close to each other in addition and subtraction problems.

4.1.2 The Performances of the Preservice Primary Teachers Outside Allocated Time

The answers of preservice primary teachers that were given after the restricted time ended were also analyzed. These answers were thoroughly examined according to the categories of the problems and the year level of the participants. The number of correct answers given by the preservice primary teachers after the given time ended was studied and given in Table 4.4 below.

Table 4.4 Preservice Primary Teachers' Number of Correct Answers After Restricted Time Ended

	Category 1*			Category 2*		Category 3*		Total
	Decoy	Far	Near	Small Distance	Large Distance	Subtraction	Addition	
Freshmen	14/27 (51.9%)	5/24 (20.8%)	11/25 (44%)	15/50 (30%)	15/26 (57.7%)	19/38 (50%)	11/38 (29%)	90/228 (39.4%)
Sophomores	7/26 (26.9%)	12/44 (27.3%)	15/40 (37.5%)	22/78 (28.2%)	12/32 (37.5%)	12/46 (26.1%)	22/64 (34.4%)	102/330 (30.9%)
Juniors	20/44 (45.5%)	3/20 (15%)	6/16 (37.5%)	17/53 (32.1%)	12/27 (44.4%)	17/46 (37%)	12/34 (35.3%)	87/240 (36.2%)
Seniors	20/43 (46.5%)	18/28 (64.3%)	14/38 (36.8%)	33/71 (46.5%)	19/38 (50%)	25/48 (52.1%)	27/61 (44.3%)	156/327 (47.7%)

*Out of 48 questions and 86 participants

According to Table 4.4. above, it was seen that more than half of the given answers after the restricted time ended were answered correctly by freshmen in decoy problems (51.9%). Moreover, a similar result was observed for large distance problems (57.7%)

in category two. Considering category three, the freshman preservice primary teachers gave correct answers to 50% of the subtraction problems after the given time ended. When the sophomores' answers given after the restricted time ended were investigated, it was seen that unlike the freshman teacher candidates, the majority of their outside restricted time answers were not correct. Otherwise, juniors achieved similar results with freshmen, even though they did not answer more than half of the problems correctly like them, 45.5% of after time ended answers in decoy problems and 44.4% of after time ended answers in large distance problems were answered correctly. Lastly, senior preservice teachers answered more than 50% of the given answers after the restricted time ended correctly, almost in every subcategory. According to the total scores, seniors had the highest percentage (47.7%) of correct answers after time expired, while sophomores had the lowest percentage (30.9%).

When the categories were examined through different levels, it was seen that except sophomores, other preservice primary teachers answered correctly to majority of the questions given after the restricted time ended. In far complement problems, senior preservice primary teachers became prominent in comparison to other preservice teachers. In near complement problems, the correct answers given after the restricted time were similar. Small and large distance problems, on the other hand, were answered correctly in nearly the same amount of time after the time limit was reached. However, small distance problems were answered slightly more correctly than large distance problems after time expired. Similar situation continued with subtraction and addition problems where subtraction problems were answered somewhat more correctly than addition problems.

The answers that were given after the restricted time ended were also analyzed according to the number of incorrect answers. The incorrect answers given by the preservice primary teachers even after the given time ended was studied and given in Table 4.5 below.

Table 4.5 Preservice Primary Teachers' Number of Incorrect Answers After Restricted Time Ended

	Category 1*			Category 2*		Category 3*		Total
	Decoy	**Far**	**Near**	**Small Distance**	**Large Distance**	**Subtraction**	**Addition**	
Freshmen	8/27 (29.6%)	4/24 (16.7%)	3/25 (12%)	8/50 (16%)	7/26 (26.9%)	8/38 (21.1%)	7/38 (18.4%)	45/228 (19.7%)
Sophomores	3/26 (11.6%)	8/44 (18.2%)	2/40 (5%)	10/78 (12.8%)	3/32 (9.4%)	10/46 (21.7%)	3/64 (4.7%)	39/330 (11.8%)
Juniors	7/44 (15.9%)	6/20 (30%)	3/16 (18.7%)	11/53 (20.8%)	5/27 (18.5%)	11/46 (23.9%)	5/34 (14.7%)	48/240 (20%)
Seniors	15/43 (34.9 %)	8/28 (28.6%)	12/38 (31.6%)	20/71 (28.2%)	15/38 (39.5%)	15/48 (31.2%)	20/61 (32.8%)	105/327 (32.1%)

*Out of 48 questions and 86 participants

According to Table 4.5. above, it was observed that majority of the given answers after the restricted time ended were answered incorrectly by freshmen in decoy problems (29.6%) and large distance problems (26.9%). In sophomores' answers that were given after the restricted time incorrectly, subtraction problems were noticeable. On the other hand, junior preservice primary teachers answered 20 far complement problems after given time ended and 30% of these problems were answered incorrectly after given time ended. Additionally, 23.9% of the subtraction problems were solved incorrectly by junior preservice primary teachers. Interestingly, different from other teacher candidates, seniors showed a high percentage of incorrectly answered problems after the restricted time ended in every subcategory. According to the total scores, seniors had the highest percentage (32.1%) of incorrect answers after time expired, while sophomores had the lowest percentage (11.8%).

The categories were also examined at different levels, and it was detected that while freshmen and seniors had high percentages of decoy problems, sophomores and juniors performed similarly. Differently, in terms of far complement problems, juniors and seniors had the greatest percentage. Besides, seniors differed from other preservice primary teachers in the number of near complement problems' answers given incorrectly after the time ended (31.6%). Senior preservice primary teachers had the highest percentage in small distance problems (28.2%), as they did in near complement problems. In addition to the seniors, freshman teacher candidates also showed a high percentage score in large distance problems and answered 26.9% of the answers given after the time ended incorrectly. In category three, while all participants answered incorrectly subtraction problems in nearly the same amount of time, only seniors were prominent in addition problems that were answered incorrectly after time expired.

Besides, correct and incorrect answers that were given after the time ended, some questions remained unanswered eve the restricted given time ended. These number of questions that could not answered by the preservice primary teachers after the given time ended was also examined and given in Table 4.6 below.

Table 4.6 Preservice Primary Teachers' Number of Unanswered Questions After Restricted Time Ended

	Category 1*			**Category 2***		**Category 3***		**Total**
	Decoy	**Far**	**Near**	**Small Distance**	**Large Distance**	**Subtraction**	**Addition**	
Freshmen	5/27 (18.5%)	15/24 (62.5%)	11/25 (44%)	27/50 (54%)	4/26 (15.4%)	11/38 (28.9%)	20/38 (52.6%)	93/228 (40.8%)
Sophomores	16/26 (61.5%)	24/44 (54.5%)	23/40 (57.5%)	46/78 (59%)	17/32 (53.1%)	24/46 (52.2%)	39/64 (60.9%)	189/330 (57.3%)
Juniors	17/44 (38.6%)	11/20 (55%)	7/16 (43.8%)	25/53 (47.2%)	10/27 (37.1%)	18/46 (39.1%)	17/34 (50%)	105/240 (43.7%)
Seniors	8/43 (18.6%)	2/28 (7.1%)	12/38 (31.6%)	18/71 (25.3%)	4/38 (10.5%)	8/48 (16.7%)	14/61 (22.9%)	66/327 (20.2%)

*Out of 48 questions and 86 participants

According to Table 4.6. above, it was seen that more than half of the given answers after the restricted time ended were remained unanswered by freshmen in far complement problems (62.5%), in small distance problems (54%) and in addition problems (52.6%). When the unanswered questions of the sophomores were investigated after the time limit expired, it was discovered that half of the problems remained unanswered in each subcategory. Furthermore, juniors achieved similar results in far complement (55%) and addition problems (50%). Additionally, juniors could not give any answer to 43.8% of near complement problems and 47.2% of small distance problems. Finally, seniors differed from other preservice teachers, they could not give any answer to 31.6% of the near complement problems that were answered after the time ended. This result of senior teacher candidates was the highest percentage among other categories. According to the total scores, sophomores had the highest percentage (57.3%) of unanswered problems after time ended, while seniors had the lowest percentage (20.2%).

When the categories were examined at different levels, it came out that only sophomores had a high percentage of decoy problems. They could not give an answer to 16 problems even after the restricted time had ended. Differently, except for the senior preservice primary teachers, all of the participants could not give an answer to more than half of the problems that were answered after the time ran out in far complement problems. In near complement problems, the percentages were close to one another; only sophomores had a slightly higher percentage. Considering the category two, every participant had high percentage except seniors in small distance problems. Notably, only sophomores had high percentage (53.1%) in large distance problems. Lastly, in category three, sophomores and juniors had a higher percentage than freshmen and seniors in subtraction problems. However, with the exception of seniors; freshman, sophomore and junior preservice primary teachers could not give any answers to the majority of the addition problems that were not answered within the restricted time.

To sum up, the responses that were given after the restricted time ended were analyzed to evaluate the performances of the participants. Firstly, the preservice primary teachers' number of correct answers given after restricted time ended were investigated. Similar to their performances within the given time, sophomores were the ones who had the lowest percentage of correct answers in comparison to other participants. According to the categories, decoy problems, large distance problems, and subtraction problems had higher percentages of correct answers than other subcategories. Secondly, the preservice primary teachers' number of incorrect answers after the restricted time ended was analyzed. It was discovered that incorrect answers were not as common as correct answers and unanswered problems. In addition, seniors were distinguished from others because they had the highest percentage in every subcategory except far complement problems, where juniors had the greatest percentage. Finally, some of the participants did not give any answer to some problems even after the restricted time ended. Therefore, these data were also analyzed and examined thoroughly. Similarly, the seniors were again exceptional and they were the ones who had noticeably the least percentage in terms of remained unanswered problems. There was not an apparent difference in category one, but the small distance problems had slightly higher percentage than large distance problems in category two. Moreover, similar with small distance problems, addition problems had somewhat greater percentage than subtraction problems. Thus, more small distance problems were remained unanswered even after the restricted time ended than large distance problems. Also, more addition problems were not given any answer after the restricted time ended than subtraction problems.

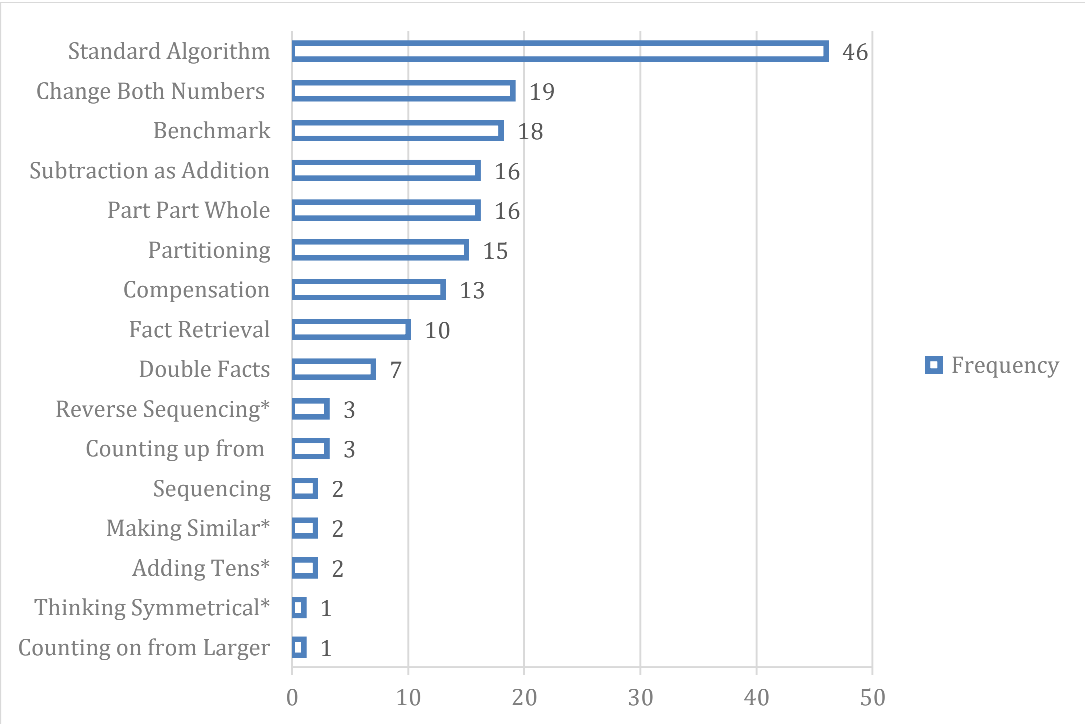
When these findings were considered altogether, decoy problems had the highest number of answers after the time ended in category one. In category two, the small distance problems were answered more times than large distance after the time limit expired. Lastly, addition and subtraction problems differed according to the different levels of the program. While addition problems had slightly more answers than subtraction after the time ended among sophomores and seniors, juniors answered more subtraction problems than addition problems after the given time ended. On the other hand, freshmen answered the same amount of addition and subtraction questions after the time ended.

4.2 The Strategies of the Preservice Primary Teachers

During the data analysis, it was also aimed to determine the strategies in the problems which preservice primary teachers answered incorrectly and could not answer in the given time. In some cases, the participants stated that they could either solve the problem with one strategy or another and explained their different solutions for those stated strategies. On the other hand, sometimes they explained one strategy explicitly. Moreover, while they were describing their one strategy solution, they used more than one strategy in the process of solving the problem with that strategy. The findings of the second research question, "For not manageable problems within the allocated time, what are the strategies of preservice primary teachers produced outside the allocated time when solving structurally-related two-digit addition and subtraction problems? were shown in this section. To answer this question, all of the used strategies were coded and analyzed according to the mental computation strategies in the literature. However, some of those strategies were not expected in the analysis process. These unexpected strategies were named and described in the methodology chapter as well. Thus, in the sections below each level of preservice primary teachers' mental computation strategies from freshman to senior levels were presented in detail. The analysis was made according to the mental computation strategies of the participants. Besides, the unexpected strategies that were described before also used to interpret the results.

4.2.1 The Freshman (Year 1) Preservice Primary Teachers' Strategies

The Figure 4.3 below shows the frequency of the freshman preservice primary teachers' strategies in solving structurally-related two-digit addition and subtraction problems.



Note. *The unexpected strategies of the PSTs

Figure 4.3 The frequency of freshman preservice primary teachers' strategies

As shown in Figure 4.3, the freshman preservice teachers stated a total of 16 strategies. Standard Algorithm (n=46, 26.1%), Change Both Numbers (n=19, 11%) and Benchmark (n=18, 10.3%) were the three most used strategies. The least used strategies were Counting on from Larger (n=1), Thinking Symmetrical (n=1), Adding Tens (n=2), Making Similar (n=2) and Sequencing (n=2). The strategies that were not expected and described in this chapter were used 8 times in total as Reverse Sequencing (n=3), Making Similar (n=2), Adding Tens (n=2), Thinking Symmetrical (n=1).

The strategies were analyzed according to the categories which were Decoy, Far, Near as Category 1, Small Distance and Large Distance as Category 2, Subtraction and Addition as Category 3.

The Table 4.7. shows the strategies of freshman preservice primary teachers investigated according to the categories.

Table 4.7 The Number of Strategies of Freshman Preservice Primary Teachers in Particular Categories

Strategies of Freshman by Categories	**Category 1**			**Category 2**		**Category 3**	
	Decoy	**Far**	**Near**	**Small Distance**	**Large Distance**	**Subtraction**	**Addition**
Standard Algorithm	10	13	23	30	16	19	27
Change Both Numbers	4	8	7	13	6	10	9
Benchmark	7	4	7	9	9	12	6
Subtraction as Addition	3	8	5	14	2	3	13
Part Part Whole	4	7	5	9	7	11	5
Partitioning	13	2	-	11	4	14	1
Compensation	2	6	5	9	4	2	11
Fact Retrieval	3	5	2	3	7	7	3
Double Facts	1	3	3	6	1	5	2
Reverse Sequencing*	3	-	-	2	1	3	-
Counting up from	-	3	-	-	3	3	-
Sequencing	2	-	-	1	1	1	1
Making Similar*	-	1	1	2	-	-	2
Adding Tens*	2	-	-	2	-	2	-
Thinking Symmetrical*	-	1	-	-	1	1	-
Counting on from Larger	-	-	1	-	1	1	-

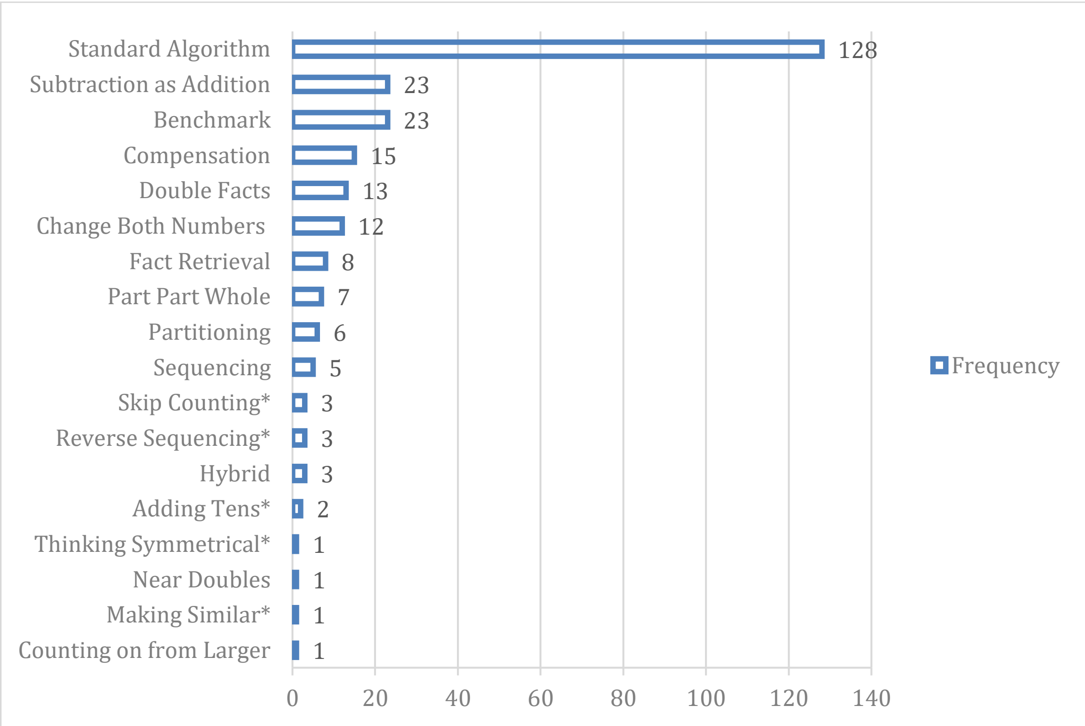
*The unexpected strategies of the PSTs

As given in the Table 4.7., the most used strategy was the Standard Algorithm, and it was mostly used in near problems, and this strategy was used the least in decoy complement problems. Besides, preservice teachers' most of the answers that was used Standard Algorithm strategy fell into in small distance, and least used in large distance problems. In addition, this strategy was mostly used in addition and least in subtraction (e.g., 48+44=92, 92-44=? Near, Small Distance, Addition problem). The second most used strategy was Change Both Numbers and it was used mostly in far problems, and least in decoy problems. Change Both Numbers was the second most popular strategy, and it was used mostly in far problems and least in decoy problems. When it comes to the second category, this strategy was used the most in small distance problems and the least in large distance problems. Moreover, it was used mostly in subtraction and least in addition (e.g., 51-26=25, 25-51=? Far, Small Distance, Subtraction problem). The third most used strategy was Benchmark, and it was used mostly equally in decoy and near complement problems, while being used the least in far complement problems. This strategy was also used equally in large distance, and in small distance problems in terms of second category. Furthermore, it was used mostly in subtraction problems and least in addition problems (e.g., 93-88=5, 93+88=? Decoy, Large Distance, Subtraction problem).

The least used strategy was Change on from Larger, and this strategy was emerged only one time in a near complement, large distance and subtraction problem which was 31-3=28, 28+3=?. Another least used strategy was Thinking Symmetrical and this strategy was seen in the 52-4=48, 48-52=? Problem, which was a far complement, large distance and subtraction problem. The Adding Tens strategy appeared two times in two different decoy, small distance and subtraction problems (e.g., 92-44=48, 92+44=?). Making Similar strategy was seen in far complement and near complement problems. Considering the category two, this strategy was seen in small distance problems and in terms of the category three this strategy was seen in addition operation (e.g., 38+46=84, 84-46=? Near Complement, Small Distance, Addition problem.) Similar with those strategies, Sequencing only occurred in decoy problems. Besides, it was seen in both small distance and large distance problems, as well as in both addition and subtraction problems (e.g., 37+44=81, 37-44=? Decoy, Small Distance, Addition problem).

4.2.2 The Sophomore (Year 2) Preservice Primary Teachers' Strategies

Figure 4.4 below shows the frequency of the sophomore preservice primary teachers' strategies in solving structurally-related two-digit addition and subtraction problems.



Note. *The unexpected strategies of the PSTs

Figure 4.4 The frequency of sophomore preservice primary teachers' strategies

As seen in the Figure 4.4, the sophomore preservice teachers stated 18 strategies in total. Standard Algorithm (n=128, 50.2%), Subtraction as Addition (n=23, 9%), Benchmark (n=23, 9%) and Compensation (n=15, 5.9%) were the three most used strategies. Counting on from Larger (n=1), Making Similar (n=1), Near Doubles (n=1), Thinking Symmetrical (n=1), Adding Tens (n=2), and Hybrid (n=3) were the least used strategies. The strategies that were not expected and described in this chapter were used 10 times in total, as Skip Counting (n=3), Reverse Sequencing (n=3), Adding Tens (n=2), Thinking Symmetrical (n=1) and Making Similar (n=1).

The strategies were also analyzed according to the categories which were Decoy, Far, Near as Category 1, Small Distance and Large Distance as Category 2, Subtraction and Addition as Category 3.

The Table 4.8. shows the strategies of sophomore preservice primary teachers investigated according to the categories.

Table 4.8 The Number of Strategies of Sophomore Preservice Primary Teachers in Particular Categories

Strategies of Sophomore by Categories	Category 1			Category 2		Category 3	
	Decoy	Far	Near	Small Distance	Large Distance	Subtraction	Addition
Standard Algorithm	35	45	48	87	41	55	73
Subtraction as Addition	5	9	9	14	9	3	20
Benchmark	7	6	10	11	12	9	14
Compensation	6	6	3	7	8	7	8
Double Facts	4	9	-	10	3	12	1
Change Both Numbers	5	4	3	8	4	9	3
Fact Retrieval	3	3	2	6	2	4	4
Part Part Whole	1	2	4	4	3	1	6
Partitioning	4	-	2	5	1	4	2
Sequencing	1	1	3	4	1	-	5
Skip Counting*	2	1	-	2	1	1	2
Reverse Sequencing*	3	-	-	1	2	3	-
Hybrid	1	-	2	2	1	1	2
Adding Tens*	2	-	-	2	-	2	-
Thinking Symmetrical*	-	1	-	-	1	1	-
Near Doubles	1	-	-	-	1	1	-
Making Similar*	-	-	1	1	-	-	1
Counting on from Larger	1	-	-	-	1	1	-

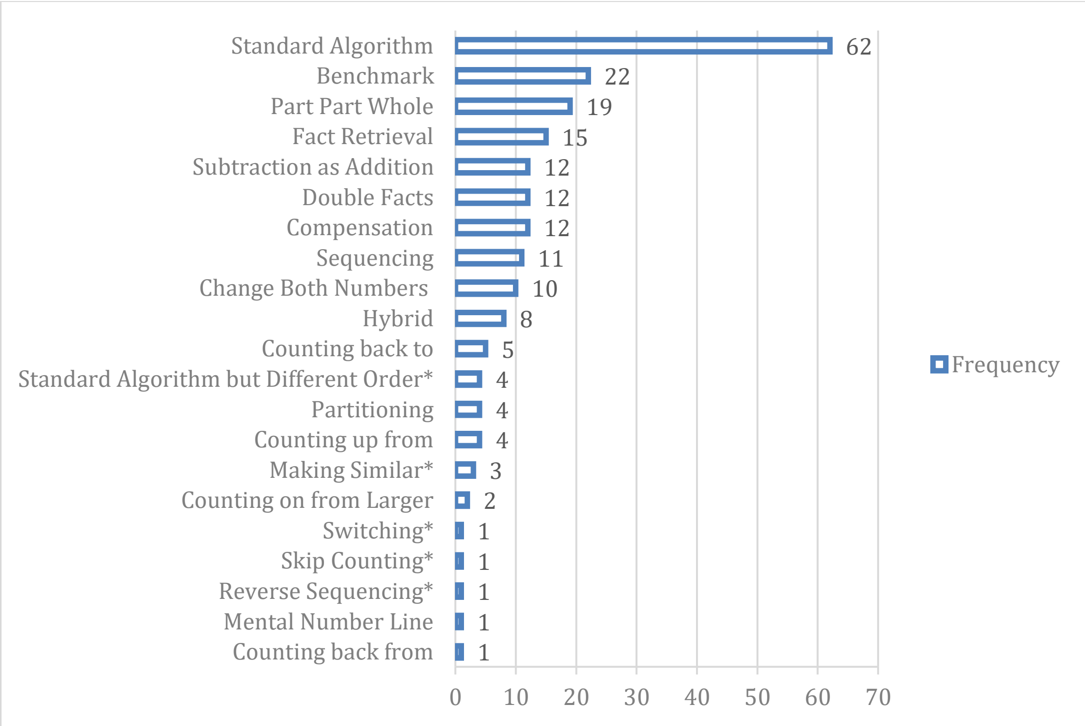
*The unexpected strategies of the PSTs

As given in the Table 4.8., the most used strategy was the Standard Algorithm, which was mostly used in near problems; this strategy was used the least in decoy complement problems. Besides, preservice teachers' most of the answers that was used Standard Algorithm strategy fell into in small distance, and least used in large distance problems. Furthermore, this strategy was mostly used in addition and least in subtraction (e.g., 44+48=92, 92-48=? Near Complement, Small Distance, Addition problem). The second most used strategy was Subtraction as Addition and it was used mostly in near and far problems, and least in decoy problems. When it comes to the second category, this strategy used the most in small distance problems and the least in large distance problems. Moreover, it was used mostly in addition and least in subtraction (e.g., 38+46=84, 84-46=? Near, Small Distance, Addition problem). The third most used strategy was Benchmark, and it was used mostly in near complement problems, least in far complement problems. This strategy was used almost equally in large distance (n=12) and small distance (n=11) problems in the second category. Furthermore, it was most commonly used in addition and least frequently in subtraction problems (e.g., 38+46=84, 84-46=? Near Complement, Small Distance, Addition problem).

The least used strategy was Change on from Larger, and this strategy was emerged only once in a decoy, large distance and subtraction problem which was 77-9=68, 77+9=?. Another least used strategy was Making Similar and this strategy was seen in the 38+46=84 84-46=? problem which was a near complement, small distance and addition problem. Near Doubles was also one of the least used strategies and it was used in the 93-88=5, 93+88=? problem by sophomores. This problem was decoy, large distance and subtraction problems. Similar with these strategies, Thinking Symmetrical strategy also came out as another least used strategy, and it was used in only far complement, large distance and subtraction problem (52-4=48, 48-52=?). Adding Tens strategy appeared two times in two different decoy, small distance and subtraction problems (e.g., 84-46=38, 84+46=?). Differently, Hybrid strategy was occurred both in near complement (n=2) and decoy (n=1), in small distance (n=2) and large distance (n=1), and in addition (n=2) and subtraction (n=1) problems (e.g., 38+46=84, 84-46=? Near Complement, Small Distance, Addition problem).

4.2.3 The Junior (Year 3) Preservice Primary Teachers' Strategies

Figure 4.5 below demonstrates the frequency of the junior preservice primary teachers' strategies in solving structurally-related two-digit addition and subtraction problems.



Note. *The unexpected strategies of the PSTs

Figure 4.5 The frequency of junior preservice primary teachers' strategies

According to Figure 4.5, the junior preservice teachers stated 22 strategies in total. Standard Algorithm (n=62, 29.5%), Benchmark (n=22, 10.5%) and Part Part Whole (n=19, 9%) were the three most used strategies. Counting back from (n=1), Mental Number Line (n=1), Reverse Sequencing (n=1), Skip Counting (n=1) and Switching (n=1) were the least used strategies. The strategies that were not expected and described in this chapter were used 10 times in total as Standard Algorithm but Different Order (n=4), Making Similar (n=3), Switching (n=1), Skip Counting (n=1) and Reverse Sequencing (n=1).

The strategies were analyzed according to the categories which were Decoy, Far, Near as Category 1, Small Distance and Large Distance as Category 2, Subtraction and Addition as Category 3.

The Table 4.9. shows the strategies of junior preservice primary teachers investigated according to the categories.

Table 4.9 The Number of Strategies of Junior Preservice Primary Teachers in Particular Categories

Strategies of Junior by Categories	Category 1			Category 2		Category 3	
	Decoy	**Far**	**Near**	**Small Distance**	**Large Distance**	**Subtraction**	**Addition**
Standard Algorithm	28	21	13	46	16	35	27
Benchmark	16	4	2	9	13	14	8
Part Part Whole	6	5	8	7	12	7	12
Fact Retrieval	8	3	4	11	4	7	8
Subtraction as Addition	4	8	-	9	3	8	4
Double Facts	4	6	2	8	4	8	4
Compensation	7	4	1	6	6	9	3
Sequencing	4	3	4	8	3	5	6
Change Both Numbers	4	4	2	8	2	7	3
Hybrid	1	3	4	7	1	1	7
Counting back to	2	2	1	2	3	2	3
Standard Algorithm but Different Order*	-	2	2	3	1	1	3
Partitioning	4	-	-	3	1	4	-
Counting up from	-	4	-	-	4	4	-
Making Similar*	2	-	1	1	2	-	3
Counting on from Larger	2	-	-	1	1	2	-
Switching*	1	-	-	-	1	1	-
Skip Counting*	1	-	-	1	-	-	1
Reverse Sequencing*	-	1	-	1	-	-	1
Mental Number Line	-	1	-	1	-	-	1
Counting back from	-	1	-	-	1	-	1

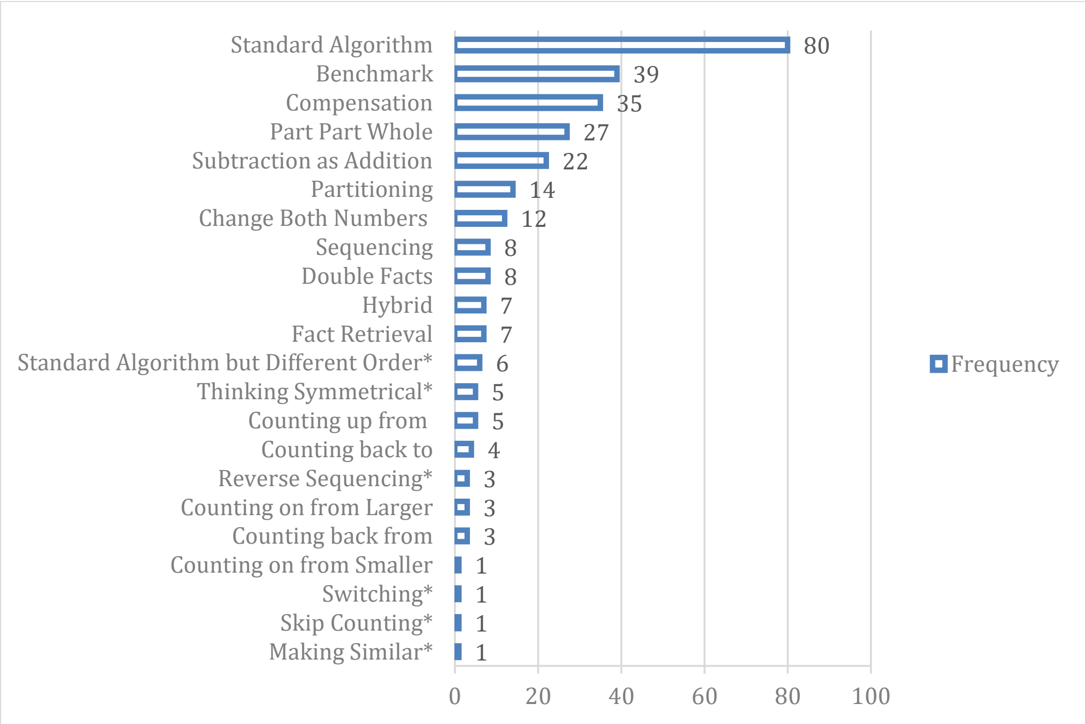
*The unexpected strategies of the PSTs

According to the Table 4.9., the most used strategy was the Standard Algorithm and it was mostly used in decoy problems, and this strategy was used the least in near complement problems. Moreover, most of the answers used the Standard Algorithm strategy by preservice teachers fell into the small distance category and were very rarely used in large distance problems. Finally, this strategy was mostly used in subtraction and least in addition (e.g., 84-38=46 84+38=? Decoy, Small Distance, Subtraction problem). The second most used strategy was Benchmark and it was used mostly in decoy problems and very little in near and far complement problems. When it comes to the second category, this strategy was used the most in large distance problems and the least in small distance problems. Moreover, it was used mostly in subtraction and least in addition (e.g., 83-79=4, 83+79=? Decoy, Large Distance, Subtraction problem). The third most used strategy was Part Part Whole, and this strategy was observed for almost the same amount of time considering the category one. In terms of the second category, the Part Part Whole strategy was used most frequently in large distance problems and least frequently in small distance problems. Furthermore, it was used mostly in addition and least in subtraction problems (e.g., 88+5=93, 93-5=? Near, Large Distance, Addition problem).

One of the least used strategy was Counting back from, and this strategy emerged only once in a far, large distance and addition problem, which was 8+26=34, 34-8=?. Another least used strategy was Mental Number Line and this strategy was seen in 18+25=43, 43-18=? Problem, which was far complement, small distance and addition problem. Reverse Sequencing was also one of the least used strategies and it was used in 25+18=43, 43-25=? problem by juniors. This problem was far complement, small distance and addition problems. Skip Counting was the other strategy that was used the least and it was used only one time in a decoy, small distance and addition problem, which was 37+44=81, 37-44=?. Similar with these strategies, Switching strategy also came out as another least used strategy and it was used in only decoy complement, large distance and subtraction problem, which was 77-9=68, 77+9=?.

4.2.4 The Senior (Year 4) Preservice Primary Teachers' Strategies

Figure 4.6 below shows the frequency of the junior preservice primary teachers' strategies in solving structurally-related two-digit addition and subtraction problems.



Note. *The unexpected strategies of the PSTs

Figure 4.6 The frequency of senior preservice primary teachers' strategies

According to Figure 4.6, the senior preservice teachers stated 22 strategies in total. Standard Algorithm (n=80, 27.4%), Benchmark (n=39, 13.4%) and Compensation (n=35, 12%) were the three most used strategies. Making Similar (n=1), Skip Counting (n=1), Switching (n=1), Counting on from Smaller (n=1), Counting back from (n=3), Counting on from Larger (n=3) and Reverse Sequencing (n=3) were the least used strategies. The strategies that were not expected and described in this chapter were used 17 times in total as Standard Algorithm but Different Order (n=6), Thinking Symmetrical (n=5), Reverse Sequencing (n=3), Switching (n=1), Skip Counting (n=1) and Making Similar (n=1). The strategies were analyzed according to the categories which are Decoy, Far, Near as Category 1, Small Distance and Large Distance as Category 2, Subtraction and Addition as Category 3.

The Table 4.10. shows the strategies of senior preservice primary teachers investigated according to the categories.

Table 4.10 The Number of Strategies of Senior Preservice Primary Teachers in Particular Categories

Strategies of Senior by Categories	**Category 1**			**Category 2**		**Category 3**	
	Decoy	**Far**	**Near**	**Small Distance**	**Large Distance**	**Subtraction**	**Addition**
Standard Algorithm	37	20	23	54	26	37	43
Benchmark	11	14	14	18	21	16	23
Compensation	12	8	15	17	18	13	22
Part Part Whole	9	11	7	8	19	12	15
Subtraction as Addition	1	16	5	11	11	10	12
Partitioning	12	2	-	8	6	12	2
Change Both Numbers	2	2	8	10	2	3	9
Sequencing	-	3	5	8	-	3	5
Double Facts	4	3	1	4	4	6	2
Hybrid	6	-	1	5	2	7	-
Fact Retrieval	5	1	1	2	5	4	3
Standard Algorithm but Different Order*	2	2	2	5	1	2	4
Thinking Symmetrical*	-	4	1	-	5	4	1
Counting up from	1	2	2	2	3	2	3
Counting back to	3	1	-	3	1	1	3
Reverse Sequencing*	1	-	2	1	2	1	2
Counting on from Larger	2	1	-	1	2	2	1
Counting back from	-	1	2	2	1	-	3
Counting on from Smaller	-	-	1	-	1	1	-
Switching*	1	-	-	-	1	1	-
Skip Counting*	1	-	-	1	-	-	1
Making Similar*	1	-	-	1	-	-	1

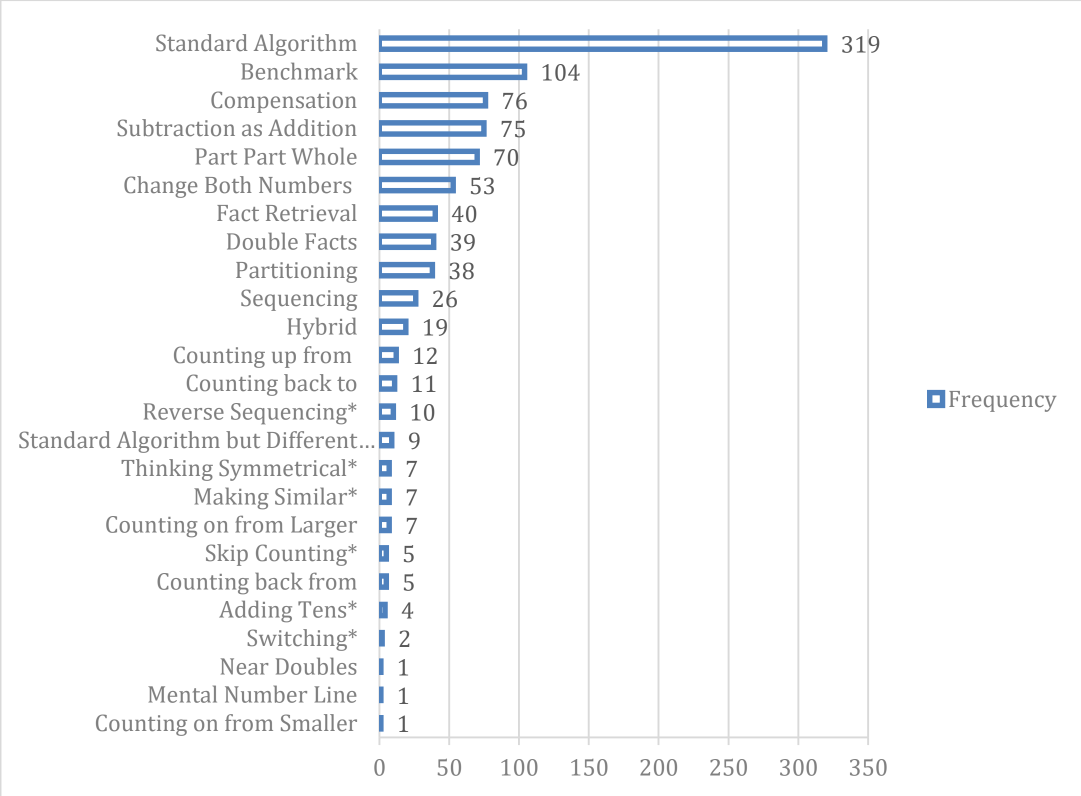
*The unexpected strategies of the PSTs

As given in the Table 4.10., the most used strategy was the Standard Algorithm and it was mostly used in decoy problems, and this strategy was used the least in far complement problems. Moreover, preservice teachers' most of the answers that was used Standard Algorithm strategy fell into in small distance, and least used in large distance problems. Finally, this strategy was mostly used in addition and least in addition (e.g., 37+44=81, 37-44=? Decoy, Small Distance, Addition problem). The second most used strategy was Benchmark and it was used mostly and equally in far complement and near complement and least in decoy problems. When it comes to the second category, this strategy was used the most in large distance problems and the least in small distance problems. Moreover, it was used mostly in addition and least in subtraction (e.g., 4+79=83 83-79=? Near Complement, Large Distance, Addition problem). The third most used strategy was Compensation, and this strategy was observed mostly in near complement problems, and least in far complement problems considering the category one. For large distance problems and small distance problems in the second category, Compensation strategy was used almost the same amount of time. Furthermore, it was used mostly in addition and least in subtraction problems (e.g., 9+68=77, 77-68=? Near Complement, Large Distance, Addition problem).

One of the least used strategy was Making Similar, and this strategy was emerged only one time in a decoy, small distance and addition problem which was 44+37=81, 44-37=?. Another least used strategy was Skip Counting and this strategy was seen in 37+44=81, 37-44=? problem which was decoy, small distance and addition problem. Switching was also one of the least used strategy and it was used in 77-9=68, 77+9=? problem by seniors. This problem was decoy, large distance and subtraction problems. Similarly, Counting on from Smaller strategy was used only in a near complement, large distance and subtraction problem (34-26=8 8+26=?).

4.2.5 The Overall Preservice Primary Teachers' Strategies

The overall strategies were shown in the Figure 4.7 below and this figure shows the frequency of the strategies of preservice primary teachers in solving structurally-related two-digit addition and subtraction problems.



Note. *The unexpected strategies of the PSTs are showed with

Figure 4.7 The overall frequency of strategies

As seen in the Figure 4.7, Standard Algorithm (n=319, 33.9%), Benchmark (n=104, 11%) and Compensation (n=76, 8.1%) were the three most used strategies. Counting on from Smaller (n=1), Mental Number Line (n=1) and Near Doubles (n=1) were the three least used strategies. The strategies that named and described before were used 53 times in total as Switching (n=2), Adding Tens (n=4), Skip Counting (n=5), Thinking Symmetrical (n=7), Making Similar (n=7), Standard Algorithm but Different Order (n=9), Reverse Sequencing (n=10) and Making Similar (n=7).

The strategies of preservice primary teachers were also analyzed according to the categories which were Decoy, Far Complement, Near Complement as Category 1, Small Distance and Large Distance as Category 2, Subtraction and Addition as Category 3.

The Table 4.11. shows the overall strategies of preservice primary teachers investigated according to the categories.

Table 4.11 The Overall Number of Strategies of Preservice Primary Teachers in Particular Categories

Strategies Overall by Categories	Category 1			Category 2		Category 3	
	Decoy	**Far**	**Near**	**Small Distance**	**Large Distance**	**Subtraction**	**Addition**
Standard Algorithm	113	100	106	217	102	149	170
Benchmark	41	30	34	50	55	52	53
Compensation	27	25	24	40	36	32	44
Subtraction as Addition	13	43	19	50	25	26	49
Part Part Whole	20	27	23	28	42	32	38
Change Both Numbers	15	18	20	39	14	29	24
Fact Retrieval	20	12	8	22	18	21	19
Double Facts	13	19	7	27	12	29	10
Partitioning	32	4	2	25	13	33	5
Sequencing	6	8	12	22	4	9	17
Hybrid	9	3	7	14	5	10	9
Counting up from	1	9	2	2	10	9	3
Counting back to	5	5	1	6	5	6	5
Reverse Sequencing*	7	1	2	5	5	7	3
Standard Algorithm but Different Order*	2	5	2	6	3	4	5
Counting on from Larger	5	1	1	2	5	6	1
Making Similar*	3	1	3	5	2	-	7
Thinking Symmetrical*	-	6	1	-	7	6	1
Counting back from	-	3	2	2	3	-	5
Skip Counting*	4	1	-	4	1	1	4
Adding Tens*	4	-	-	4	-	4	-
Switching*	2	-	-	-	2	2	-
Counting on from Smaller	-	-	1	-	1	1	-
Mental Number Line	-	1	-	1	-	-	1
Near Doubles	1	-	-	-	1	1	-

*The unexpected strategies of the PSTs

As given in the Table 4.11., the most used strategy was the Standard Algorithm and it was mostly used in decoy problems, and this strategy was used the least in far complement problems. Besides, preservice teachers' most of the answers that was used Standard Algorithm strategy fell into small distance, and least used in large distance problems. In addition, this strategy was mostly used in addition and least in subtraction problems (e.g., 44+37=81, 44-37=? for Decoy, Small Distance, Addition problem). The second most used strategy was Benchmark and it was used mostly in decoy problems, and least in far complement problems. When it comes to the second category, this strategy was used the most in large distance problems and the least in small distance problems. Moreover, it was used almost the same amount in addition and subtraction (e.g., 68+9=77, 68-9=? for Decoy, Large Distance, Addition problem). The third most used strategy was Compensation, and it was used mostly in near complement problems, least in decoy problems. This strategy was also used mostly in small distance, and least in large distance problems in terms of second category. Furthermore, it was used mostly in subtraction problems and least in addition problems (e.g., 43-25=18, 18+25=? Near Complement, Small Distance, Subtraction problem).

The least used strategy was Near Doubles, and this strategy emerged only one time in a decoy, large distance and subtraction problem, which was 93-88=5, 93+88=?. Another least used strategy was Mental Number Line and this strategy was explicitly seen in the 18+25=43, 43-18=? problem, which was a far complement, small distance and addition problem. Similar with those strategies, Counting on from Smaller occurred in near complement, large distance and subtraction (34-26=8, 8+26=?) problem only one time.

To sum up, all four levels of preservice primary teachers' mental computation strategies were investigated. It was seen that some preservice teachers unexpected their strategies (e.g., Making Similar, Switching, etc.). These strategies were applied occasionally. On the other hand, the three most used strategies were identified across all levels of participants. The three most commonly used strategies were depicted in Figure 4.8 below.

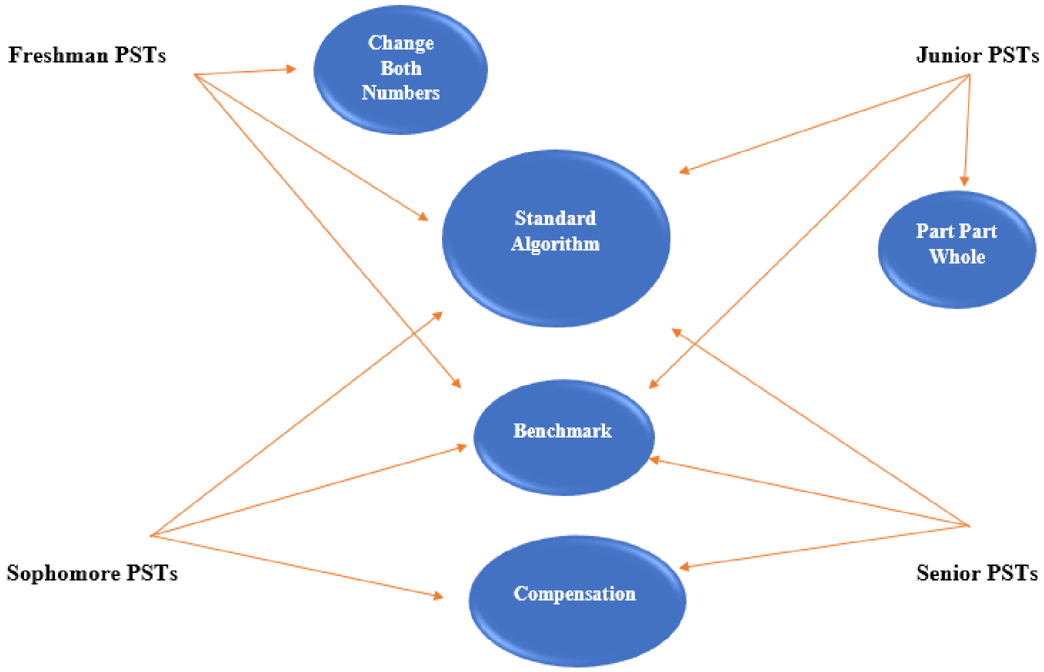


Figure 5.8 The three most used mental computation strategies

According to Figure 4.8, Standard Algorithm and Benchmark strategies were used by all four levels of the preservice primary teachers. These strategies were among three most used strategies in freshman, sophomore, junior and seniors. Beyond these two strategies, freshman preservice teachers used the Change Both Numbers strategy dominantly. On the other hand, junior preservice primary teachers used Part Part Whole strategy. Finally, sophomore and senior preservice teachers chose to employ the Compensation strategy the most, besides the Standard Algorithm and Benchmark strategies.

The data were also examined according to the categories. These categories were Category 1 (Decoy, Far Complement, Near Complement), Category 2 (Small Distance, Large Distance) and Category 3 (Subtraction, Addition). The most used mental computation strategies were determined according to the categories and given in the below Figure 4.9.

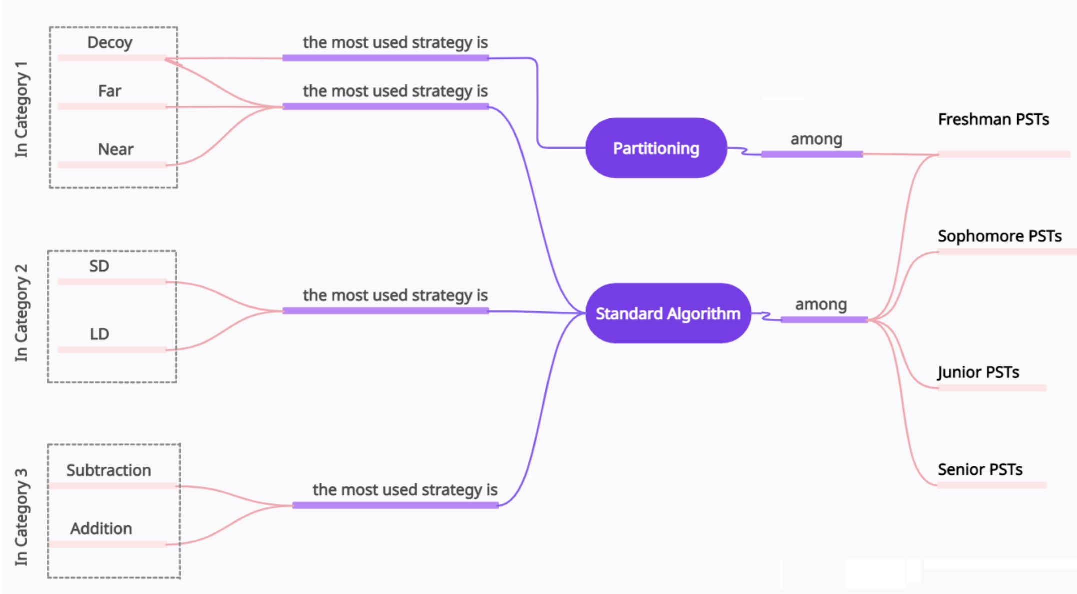


Figure 4.9 The most used strategies according to the categories

As seen in the Figure 4.9, all levels except freshman preservice teachers employed the Standard Algorithm strategy the most under each category. Differently, freshmen preservice primary teachers utilized Partitioning strategy in Decoy problems dominantly. However, similar with other levels, they also used the Standard Algorithm in all other categories. As a result, the most frequent used strategies are Standard Algorithm and Partitioning strategies according to the categories.

4.2.6 The Variety of Mental Computation Strategies

The findings of the research were analyzed and strategies were investigated in the previous section. According to the Figure 4.3, Figure 4.4, Figure 4.5 and Figure 4.6, the variety of the strategies were examined. Table 4.12. below shows the number of the different strategies that were used by preservice primary teachers' in solving structurally-related two-digit addition and subtraction problems.

Table 4.12 The Number of Different Strategies

	Freshman	Sophomore	Junior	Senior
Number of Different Strategies	16	18	22	22

*Out of 48 questions

As seen in Table 4.12, the variety of strategies increases from freshmen to seniors. While freshman preservice primary teachers were using 16 different strategies, the sophomore preservice primary teachers used 18 different strategies. Furthermore, it could be inferred that junior and senior preservice teachers used more strategy than freshman and sophomore preservice teachers since both juniors and seniors used 22 different strategies while solving structurally-related addition and subtraction problems.

4.3 Strategies across Problem Categories

In this section, the answers and strategies were investigated according to the categories. The first category was the complement category, and decoy, far complement, and near complement problems were the sub-categories. In this category there were 16 problems under each complement type. The second category was the distance category, and small distance and large distance problems were the sub-categories of this second category and each distance type included 24 problems. Finally, the third category was the operation category, and subtraction and addition problems were the sub-categories of this final category, and each sub-category contained 24 problems. Therefore, in this section the answers were investigated in terms of these categories and compared between the different years of the preservice primary teachers.

4.3.1 Strategies in Category 1

The Category 1 contained Decoy, Far Complement and Near Complement problems and there were 16 problems under each sub-category. As mentioned before, the data analysis of the strategies in the problems which preservice primary teachers answered incorrectly and could not answer in the given time were determined. The participants sometimes stated that they could either solve with one strategy or another and explained their solutions. On the contrary, sometimes they stated one strategy explicitly and while they were describing their one strategy solution, they used more than one strategy in the process of solving with that strategy. Thus, there could be more than one strategy usage in one problem. The analyzes were made and the strategies of preservice primary teachers were investigated in accordance with the Category 1 (Decoy, Far Complement, Near Complement) and Table 4.19. was prepared.

The Table 4.13. below shows the strategies according to the category one.

Table 4.13 Strategies according to the Category 1

Strategies	**Freshman**			**Sophomore**			**Junior**			**Senior**		
	Decoy	**Far**	**Near**	**Decoy**	**Far**	**Near**	**Decoy**	**Far**	**Near**	**Decoy**	**Far**	**Near**
Standard Algorithm	10	13	23	35	45	48	28	21	13	37	20	23
Benchmark	6	4	7	7	6	10	16	4	2	11	14	14
Compensation	2	6	5	6	6	3	7	4	1	12	8	15
Subtraction as Addition	3	8	5	5	9	9	4	8	-	1	16	5
Part Part Whole	4	7	5	1	2	4	6	5	8	9	11	7
Change Both Numbers	4	8	7	5	4	3	4	4	2	2	2	8
Fact Retrieval	3	5	2	3	3	2	8	3	4	5	1	1
Double Facts	1	3	3	4	9	-	4	6	2	4	3	1
Partitioning	13	2	-	4	-	2	4	-	-	12	2	-
Sequencing	2	-	-	1	1	3	4	3	4	-	3	5
Hybrid	-	-	-	1	-	2	1	3	4	6	-	1
Counting up from	-	3	-	-	-	-	-	4	-	1	2	2
Counting back to	-	-	-	-	-	-	2	2	1	3	1	-
Reverse Sequencing*	3	-	-	3	-	-	-	1	-	1	-	2
Standard Algorithm but Different Order*	-	-	-	-	-	-	-	2	2	2	2	2
Counting on from Larger	-	-	1	1	-	-	2	-	-	2	1	-
Making Similar*	-	1	1	-	-	1	2	-	1	1	-	-
Thinking Symmetrical*	-	1	-	-	1	-	-	-	-	-	4	1
Counting back from	-	-	-	-	-	-	-	1	-	-	1	2
Skip Counting*	-	-	-	2	1	-	1	-	-	1	-	-
Adding Tens*	2	-	-	2	-	-	-	-	-	-	-	-
Switching*	-	-	-	-	-	-	1	-	-	1	-	-
Counting on from Smaller	-	-	-	-	-	-	-	-	-	-	-	1
Mental Number Line	-	-	-	-	-	-	-	1	-	-	-	-
Near Doubles	-	-	-	1	-	-	-	-	-	-	-	-

*The unexpected strategies of the PSTs

As given in the Table 4.13., in Decoy problems, while freshman preservice primary teachers were using Partitioning strategy the most, all the other preservice primary teachers carried out Standard Algorithm. PST1.17 explained the partitioning strategy solution for a decoy problem (84-38=46, 84+38=?) with *"First I add 80 and 30 to make a short cut, then it turns out to be 110, then 8 and 4 are 122 from here."* They stated that partitioning strategy seems to be a short cut for this problem. For the same problem another freshman PST1.6 described partitioning strategy as *"First I thought of this (84) as an integer 80, then this (38) as 30 exactly and add 80 and 30 and then add the ones digits and add them on top. I add 80 to 30. 110. Then I add 8 to 4. 12. I add 12 to 110."* The freshmen participants also applied standard algorithm like the other preservice primary teachers. For example, for the same decoy problem PST1.8 declared *"Here again, I am adding 4 and 8 directly under the other 12, here is I have 1 in my hand. It is 112. I add 8 to 3 first and then add what is in my hand."* They explicitly stated standard algorithm strategy. When the strategies of sophomores, juniors, and seniors were being considered, they mostly used the standard algorithm for every decoy problem. One junior participant PST3.17 described the solution process *"For some reason, I thought I have something on my hand, I made a mistake with it first. I can add directly, in fact, 9, 4 more 3, 136 in a row."* in a decoy problem (92-44=48, 92+44=?). Here, as we can understand, the participant found the answer false because while applying the standard algorithm strategy they thought they had one ten extra and added wrongly.

For instance, in another decoy problem (83-79=4, 83+79=?), PST4.24 utilized both Compensation and Double Facts. Their explanation was *"I think of 79 as 80. 8, 8, 16. That's what I do, ma'am. I say 80,80 160. Then I add up the remainder so 163 and subtract 1 from there. This (79) is not 80. I'm taking out 1 extra."* This participant thought 83+79 as 80+80 and found 160. They added the second addend with one extra and subtracted that extra from the final result which was 163. PST4.16 stated for the decoy problem 84-38=46, 84+38=? *"So at first I compute in my mind. I round 84 to 80 and 38 is rounded to 30. I add 20 to 80, I add 10 to it. 110. I add 4 units to 8 units so they are 12 units, I find it so. Then added 12 to 110, 122."* Here the participant used partitioning strategy, added tens first then ones, and finally added altogether. While finding tens, PST4.16 rounded the numbers so that they could reach the benchmark. After adding tens, the participant showed the part part whole relation because 80+30 is also 80+20+10.

Besides, when we look for some of the least used strategies in the Decoy problems, Counting back to, Making Similar and Switching strategies were only seen in juniors and seniors. On the other hand, Adding Tens strategy is only observed in freshmen and sophomores.

According to the Table 4.13., in terms of Far Complement problems, all of the participants applied Standard Algorithm strategy the most. PST2.19 explained the standard algorithm strategy solution for a far complement problem (26+25=51 51-26=?) with *"I tried to subtract 6 from 11 and find them one under the other. Then subtract 2 from 4. Because doing it that way feels more comfortable, even if I can't do it fast, it feels more comfortable to do it in my head."* They emphasized that standard algorithm strategy makes the participant comfortable with the problem while doing mental computation. For the same problem another sophomore PST2.21 described *"Here, too, I tried to subtract 6 from 11 and find 5, and 2 from 4 to find 2, but I subtracted the smaller from the larger. I subtracted 1 from 6, 2 from 5 and found 35."* Here it can be inferred that while doing standard algorithm mentally in a restricted time period, there could be misconceptions like subtracting from larger instead of borrowing or trading. Furthermore, while freshmen, sophomores and juniors were using standard algorithm dominantly in far complement problems, seniors used the other strategies too. For example, PST4.3 stated their solution for a far complement problem (25+18=43, 43-25=?) as *"Here I added 5 to 25, it became 30, then I needed 13 for 43, I added 5 to it and said 18. I tried to complete 25 to 43."* They explicitly carried out subtraction as addition strategy. Another example for the 26+8=34, 34-26=? far complement problem, PST2.4 used both Subtraction as Addition and Benchmark strategies and explained their solution as *"It was because of stress that the given time was not enough. It takes 4 to round this (26) up to 30. So I'm guessing it's already 4. On top of that, I add the 4 here, 30 to 34, I add 8. If I looked at the first given operation, there is already an answer. I've never looked."* The participant used subtraction as addition strategy and while applying that strategy they used 30 as a benchmark.

Some of the least used strategies in the Far Complement problems were examined, Counting back to, Counting back from, Standard Algorithm but Different Order strategies were only seen in juniors and seniors. This result was similar with decoy problems. On the other hand, Making Similar strategy was only observed in freshmen. Also, Hybrid strategy was only used by juniors when the far complement problems were considered. Differently, Thinking Symmetrical strategy was observed in every preservice primary teacher except juniors.

In accordance with the Table 4.13., in term of Near Complement problems, similar with far complement problems, all of the preservice primary teachers used Standard Algorithm strategy the most. PST1.19 declared the standard algorithm strategy solution for a near complement problem (48+44=92, 92-44=?) with *"Here I would do normal subtraction. -What do you mean by normal subtraction? Here's 92 and 44 (the participant write in air with their finger one below another) I'm subtracting 4 from 12 with trading."* They emphasized that standard algorithm strategy is the normal subtraction. For the same problem another freshman PST1.11 described *"Let me do it again quickly. From 12, 4 subtracted so 8. The number on the right (9), that is, the number in the tens place, became 8. Because I went trading, 8 left. 4 subtracted from 8, 4. 48. I forgot to drop the ten again from here."* Here it is observed that this participant executed standard algorithm but forgot they traded one ten with ten ones. While doing standard algorithm mentally in a restricted time period, it was observed that there could be misconceptions like forgetting the traded number.

When it was investigated for some of the least used strategies in the Near Complement problems, Standard Algorithm but Different Order strategy was seen both in juniors and seniors. This result was similar with decoy problems. Different from decoy problems, Partitioning strategy was only observed in sophomores. In addition to these results, Sequencing and Hybrid strategies were emerged from everyone except freshmen. Contrarily, Making Similar strategy was used by everyone except seniors. On the other hand, Counting on from Larger strategy was only observed in freshmen. Using these strategies in time restricted situations could be effective for them.

4.3.2 Strategies in Category 2

The category two included Small Distance and Large Distance problems and there were 24 problems under each sub-category. As stated earlier, the strategies in the problems which preservice primary teachers answered incorrectly and could not answer in the given time were determined. The participants sometimes stated that they could either solve with one strategy or another and explained their solutions. On the other hand, sometimes they stated one strategy explicitly and while they were describing their one strategy solution, they used more than one strategy in the process of solving with that strategy. Therefore, there could be more than one strategy usage in one problem.

The data were analyzed and the strategies of preservice primary teachers were investigated in Category 2 (Small Distance, Large Distance) and Table 4.14. was presented.

The Table 4.14. below shows the strategies according to the category two.

Table 4.14 Strategies according to the Category 2

Strategies	Freshman		Sophomore		Junior		Senior	
	Small Distance	Large Distance	Small Distance	Large Distance	Small Distance	Large Distance	Small Distance	Large Distance
Standard Algorithm	30	16	87	41	46	16	54	26
Benchmark	9	9	11	12	9	13	18	21
Compensation	9	4	7	8	6	6	17	18
Subtraction as Addition	14	2	14	9	9	3	11	11
Part Part Whole	9	7	4	3	7	12	8	19
Change Both Numbers	13	6	8	4	8	2	10	2
Fact Retrieval	3	7	6	2	11	4	2	5
Double Facts	6	1	10	3	8	4	4	4
Partitioning	11	4	5	1	3	1	8	6
Sequencing	1	1	4	1	8	3	8	-
Hybrid	-	-	2	1	7	1	5	2
Counting up from	-	3	-	-	-	4	2	3
Counting back to	-	-	-	-	2	3	3	1
Reverse Sequencing*	2	1	1	2	1	-	1	2
Standard Algorithm but Different Order*	-	-	-	-	3	1	5	1
Counting on from Larger	-	1	-	1	1	1	1	2
Making Similar*	2	-	1	-	1	2	1	-
Thinking Symmetrical*	-	1	-	1	-	-	-	5
Counting back from	-	-	-	-	-	1	2	1
Skip Counting*	-	-	2	1	1	-	1	-
Adding Tens*	2	-	2	-	-	-	-	-
Switching*	-	-	-	-	-	1	-	1
Counting on from Smaller	-	-	-	-	-	-	-	1
Mental Number Line	-	-	-	-	1	-	-	-
Near Doubles	-	-	-	1	-	-	-	-

*The unexpected strategies of the PSTs

As given in the Table 4.14., in Small Distance problems, various different strategies came out in small distance problems but all of the preservice primary teachers applied Standard Algorithm strategy the most. PST2.18 explained the standard algorithm strategy solution for a small distance problem (18+25=43 43-18=?) with *"I tried to do direct subtraction. Normally as taught one under the other. I'm subtracting 8 from 3. I'm going to my neighbor. Like that."* They emphasized that standard algorithm is the direct method and it is what is taught at school. For the same problem one junior participant PST3.19 clarified *"Here is another problem that I don't have enough time to break a ten. From 3..., since 3 isn't enough, I'm breaking tens. I subtract 8 from 13 5. There are 3 left. I subtract 1 from 3."* They declared that trading process takes time while doing standard algorithm. Moreover, PST2.19 used standard algorithm and made an explanation as *"I probably had a hard time doing it mentally. If I had a pen and paper, I would do it with a pen and paper. I'd subtract 29 from 53. 9 out of 13 4. 2 out of 4 2. Bottom up. Then I would add minus."* This participant expressed that they had a hard time making mental computation and even they revealed their reliance on materials like pen and paper so that they could implement the operation.

For another small distance problem example (44+48=92, 92-48=?), PST4.16 stated *"90, well if I say 100 here. So there are 100 minus plus 8's. If I add 8 here, it's 56. 100-56, I subtracted 50 from 100, 50. I subtracted 6 from 50 44."* It could be understood this participant carried out change both numbers, double facts and sequencing strategies for a small distance problem. Firstly, they changed 92 to 100 and 48 to 56 to keep the balance, used 100-50=50 fact, and subtracted tens first, then subtracted ones finally.

Apart from this, some of the least used strategies were examined in the Small Distance problems, Counting back to and Standard Algorithm but Different Order strategies were only seen in juniors and seniors. On the other hand, like decoy problems, Adding Tens strategy was only observed in freshmen and sophomores. In addition to these results, Skip Counting and Hybrid strategies were emerged from everyone except freshmen.

According to the Table 4.14., in Large Distance problems, several different strategies used in large distance problems but all of the preservice primary teachers utilized Standard Algorithm strategy the most. Interestingly, even in large distance problems like 28+3=31, 31-28=? some participants used standard algorithm strategy. For example, one senior participant PST4.19 executed the standard algorithm strategy solution for that problem with *"It's actually a very easy operation. Again, I subtracted 8 from 11. I subtracted 2 from 2. One down below another."* They emphasized that this problem is not a difficult problem and yet they chose to apply standard algorithm. Another freshman PST1.9 made an explanation for the same problem *"I subtract 8 from 11, then 2 remains, after 2 subtract 2 it's already 0."* and also executed standard algorithm. For another large distance problem (26+8=34, 34-26=?) PST2.20 described that *"I have a board in my head, I'm writing 34. Then I write 26 one below the other. It just happens that way. For example, some people are calculating with a shopkeeper arithmetic. In my head, I directly draw with a board marker 4 14. I subtract 6 from 14. 8. Subtract 2 from 2 then 0. The answer is 08."* After they were asked what they meant by saying shopkeeper arithmetic, they added that *"I actually learned while I was working. There, my boss was calculating faster than me. Because the shopkeeper arithmetic is faster than our operation. He told me how we were doing it... Even, my father calculates it the same way. I can't do that. I can't remember that way. I do something like, even if I write it wrong in my head, I delete it with my hand. I imagine that."* According to the PST2.20, there were different ways to solve a problem; one of them was shopkeeper arithmetic, and this was the faster way, but the participant could not internalize that they couldn't remember the solving process. The other option was the standard algorithm, and even when another person attempted to teach the participant faster ways, they followed the standard algorithm strategy. Moreover, similar with PST2.19, PST2.20 expressed the need for paper and pencil or mentally imagining it.

Meanwhile, some of the least used strategies were also inspected and Counting back to, Counting back from, Standard Algorithm but Different Order and Switching strategies were only seen in juniors and seniors in terms of the Large Distance problems. On the other hand, Sequencing was observed in everyone except seniors, Hybrid strategy was applied by everyone except freshmen, where everyone except juniors applied Reverse Sequencing strategy. Differently, Making Similar was only seen in juniors' strategies.

4.3.3 Strategies in Category 3

The category three contained Subtraction and Addition problems and there were 24 problems under each sub-category. As mentioned earlier, the strategies in the problems which preservice primary teachers answered incorrectly and could not answer in the given time are determined. The participants sometimes stated that they could either solve with one strategy or another and explained their solutions. On the contrary, sometimes they stated one strategy explicitly and while they were describing their one strategy solution, they used more than one strategy in the process of solving with that strategy. Thus, there could be more than one strategy usage in one problem.

The strategies of preservice primary teachers were investigated in Category 3 (Subtraction, Addition) and Table 4.15. was presented.

The Table 4.15. below shows the strategies according to the category three.

Table 4.15 Strategies according to the Category 3

Strategies	Freshman		Sophomore		Junior		Senior	
	Subtraction	Addition	Subtraction	Addition	Subtraction	Addition	Subtraction	Addition
Standard Algorithm	19	27	55	73	35	27	37	43
Benchmark	12	6	9	14	14	8	16	23
Compensation	2	11	7	8	9	3	13	22
Subtraction as Addition	3	13	3	20	8	4	10	12
Part Part Whole	11	5	1	6	7	12	12	15
Change Both Numbers	10	9	9	3	7	3	3	9
Fact Retrieval	7	3	4	4	7	8	4	3
Double Facts	5	2	12	1	8	4	6	2
Partitioning	14	1	4	2	4	-	12	2
Sequencing	1	1	-	5	5	6	3	5
Hybrid	-	-	1	2	1	7	7	-
Counting up from	3	-	-	-	4	-	2	3
Counting back to	-	-	-	-	2	3	1	3
Reverse Sequencing*	3	-	3	-	-	1	1	2
Standard Algorithm but Different Order*	-	-	-	-	1	3	2	4
Counting on from Larger	1	-	1	-	2	-	2	1
Making Similar*	-	2	-	1	-	3	-	1
Thinking Symmetrical*	1	-	1	-	-	-	4	1
Counting back from	-	-	-	-	-	1	-	3
Skip Counting*	-	-	1	2	-	1	-	1
Adding Tens*	2	-	2	-	-	-	-	-
Switching*	-	-	-	-	1	-	1	-
Counting on from Smaller	-	-	-	-	-	-	1	-
Mental Number Line	-	-	-	-	-	1	-	-
Near Doubles	-	-	1	-	-	-	-	-

*The unexpected strategies of the PSTs

According to the Table 4.15., in Subtraction problems, many different strategies used in subtraction problems but all of the preservice primary teachers applied Standard Algorithm strategy the most. PST3.15 declared the standard algorithm strategy solution for a subtraction problem (84-38=46, 84+38=?) with *"So I collected it normally. 8 and 4, 12. So in the form of addition with trading like one under the other. 8 and 4 is 12. 1 in hand. 9. -9 and 3 is 12."* Another example is that PST2.18 clarified their thinking for a subtraction problem (84-46=38 84+46=?), *"I did the normal addition. I normally collect them one after the other. 4 and 6 10. 10 had 0 in the hand 1. 8 and 4 more 12, I had 1 in the hand 13. 130."* These participants both emphasized that standard algorithm is the normal strategy.

Another example for a subtraction problem (83-79=4, 83+79=?), PST2.3 explained their solution with *"I thought 79 was 80. I added 80 with 80. 160. I gave 2 from there. 162."* This participant used compensation strategy and thought 83+79 as 83+80, then applied double facts and found 80+80, 160 then added 2 to reach the answer. Moreover, the same participant for another subtraction problem *"I always used the same strategy here, but I collected it wrong. From here (93) I give 2 and complete 88 to 90. So I'm actually completing the close one. 88 seems to be difficult to add up like this, so I add 2 from 93 to 88, 90. Then, 90 and 90, 180. 181."* This participant carried out Change Both Numbers strategy and altered 93+88 to 91+90. Then, they used Double Facts and found 90+90 and added 1 more for a subtraction problem.

Further, some of the least used strategies in the Subtraction problems were investigated, Counting back to, Standard Algorithm but Different Order and Switching strategies were only seen in juniors and seniors. On the contrary, like decoy and small distance problems Adding Tens strategy was only observed in freshmen and sophomores. In addition to these results, Hybrid strategy was emerged from everyone except freshmen where Reverse Sequencing was seen everyone except juniors.

As given in the Table 4.15., in Addition problems, numerous different strategies used in subtraction problems but all of the preservice primary teachers executed Standard Algorithm strategy the most. PST4.23 described the standard algorithm strategy solution for an addition problem (44+37=81 44-37=?) with *"I thought about this one* *under the other, I went to the ones place, I thought if 14 is 7, what would be left, I thought 7, I have 3, 3 out of 3, so the answer is 7."* For another addition problem 26+25=51, 51-26=? PST4.20 explained their standard algorithm solution by *"There will be 5 here, I did 11-6. Then 1 from here went, 2 from 4-2 and the answer is 25."*

Another example was that for the problem 48+44=92, 92-44=?, PST4.19 clarified *"92-44. For example, I would say 90-45. I find 45 directly. Then I counted 2 as missing. 47. I also overcounted this one. 48."* The participant changed both numbers and also used double facts. First, they made 92-44, 90-45. Then, they used double facts and reached 45. Finally, they added the extras. For the same problem PST4.24 stated *"This is what happens because I actually push a little too hard. For example, I would consider 90 to 45. That would be 45. But now it's 2 more. Are we reducing? That's why I am confusing 2 here and 1 here. So I'd subtract 4 from 12 and 4 from 8, it is 48."* This participant tried to solve with the same way but got confused and changed their strategy to standard algorithm. They specifically said that by trying Change Both Numbers strategy, they pushed their limits and got confused. Then, they chose to calculate with standard algorithm, it could be inferred that standard algorithm is much easier or more comfortable for them.

When it comes to some of the least used strategies in the addition problems, similar with other categories, Counting back to, Reverse Sequencing, Standard Algorithm but Different Order and Counting back from strategies were only seen in juniors and seniors. It could be interpreted by the application course that they took during the third and fourth years of the program. On the contrast, Hybrid strategy was only appeared in sophomores and juniors and Skip Counting strategy was observed in everyone except freshmen.

In summary, this chapter presented the findings of the study that explored primary preservice teachers mental computation strategies in part part whole related addition and subtraction problems in the following aspects: 1) the number of correct answers of PSTs within the given time, 2) the number of correct answers of PSTs outside the given time, 3) performances across categories of the problems, 4) performances across level of teacher education program (i.e. freshman, sophomore, junior, senior), 5) strategies across categories of the problems, 6) strategies across level of teacher education program (i.e. freshman, sophomore, junior, senior), 7) variety of the strategies, 8) range of strategies across problem characteristics. In the next chapter those findings were discussed and main conclusions were presented.

CHAPTER 5

CONCLUSION AND DISCUSSION

This chapter includes conclusion and discussion of the study. Firstly, the results of the study will be discussed. Then, some limitations will be explained, finally study's recommendations and implications for further studies will be made.

5.1 Discussions

Mental computation abilities are the competence that mathematics educators want to develop in children, and to see whether preservice teachers have this competence or not is a significant issue that needs to be investigated because preservice teachers are the future teachers who will educate children. Therefore, this study focuses on preservice teachers and examines their number sense and mental computation abilities. The purpose of the related study is to investigate the mental strategies of preservice primary teachers while computing two-digit addition and subtraction problems that are related in terms of the part part whole structure. In this section, the preservice primary teachers' performances, their fluency in mental computation process and the used mental strategies will be discussed and compared with the current body of literature.

5.1.1 The Performances of Preservice Primary Teachers

The first research question of the study was "*What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems?"* There were also two sub-research questions related with the first research question. The first one was "*What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems within the allocated time?"* and the second question was *"What are the performances of* *preservice primary teachers when solving structurally-related two-digit addition and subtraction problems outside the allocated time?"* To answer these research questions, I examined primary preservice teachers' correct and incorrect responses within the allocated time, and correct and incorrect responses given outside the allocated time. Also, I examined the questions that remained unanswered even after the restricted time ended. In this section, findings regarding the first sub-research question will be addressed and the performances based on the correct answers within the allocated time will be discussed. The findings for this research question showed that the accurate responses of preservice primary teachers in the given time was 3387 and this score corresponded to 82% of overall answers. This percentage revealed that accuracy was not a significant problem for the primary teacher candidates and most of the answers were given correctly. The performances of preservice primary teachers were examined and this study found that the most successful level was freshman preservice primary teachers (85%) when it comes to the giving correct answers in the restricted time. The juniors and seniors were following them with 83.5% and 81.8% success, respectively. However, the lowest achievement belonged to the sophomores (78.1%). This result was somewhat similar to Yaman's (2015) study, which was conducted with preservice primary teachers. The researcher found that juniors and seniors had the highest achievement. Especially, it was seen in that study, the number sense performances of freshmen, sophomores and juniors increased as the grade level increases. Moreover, it was revealed that seniors' scores were close to the juniors' performances. The researcher explained this result by referring to Teaching Mathematics I and Teaching Mathematics II courses that could affect juniors and seniors positively. In the light of this study, this situation could explain why seniors and juniors were more successful than sophomores. However, this reasoning was not suitable for explaining freshman preservice primary teachers' success. We could explain this diversity with different experiences in earlier school life, the recent university entrance exam experience and individual mathematics competencies.

This study's data collection tool was combined with two different studies so the findings were also discussed according to the results of these studies wherever it is possible. When Peters et al.'s (2010) predictions about the categories that were investigated were considered, they concluded that large distance (e.g., 71-2=?) problems could be answered more accurately. In overall findings, this claim was consistent with the current research, while 85.6% of the large distance problems were answered correctly, 78.5% of the small distance problems were answered correctly. In terms of year levels, freshman teacher candidates answered 88.5% of large distance problems correctly and 81.4% of small distance problems, sophomore teacher candidates answered 83% of large distance problems correctly and 73.2% of small distance problems, junior teacher candidates answered 87.2% of large distance problems correctly and 79.8% of small distance problems, senior teacher candidates answered 84% of large distance problems correctly and 78.7% of small distance problems. To summarize, this situation existed across all levels, and large distance problems were solved more successfully. According to Peters et al. (2010), this was addressed with the process of strategy selection. Since the strategy selection procedure was less apparent when dealing with small distance problems, the problem-solving process could take a long time and result in more mistakes than with large distance problems.

Besides overall scores, to understand the number sense abilities of preservice primary teachers, it was also required to look for indicators. According to the studies about number sense, the individuals with developed number sense were expected to understand the meaning of the numbers well, develop multiple bonds between numbers and make mental computation flexible (NCTM, 1989). Additionally, they were supposed to recognize that there are different ways to arrive at a solution rather than just following the rules (Howden, 1989). Therefore, the individual with developed number sense should be aware of the connections between operations and have operation sense (Bresser & Holtzman, 1999; Slavit, 1999). Since this study was related with addition and subtraction problems that contained part part whole relations, it was expected that the participants understand those problem structures, recognize fact families, use these part part whole relations, interpret the shortcuts and apply subtraction as addition strategy accordingly. Anyhow, most of the preservice primary teachers did not recognize that relationship and the problem structure and did not employ subtraction as addition strategy by using shortcuts, they also applied standard algorithm strategy dominantly.

5.1.2 Fluency in Mental Computation

The first research question of this study was *"What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems?"* To answer this research question, besides correct answers within the restricted time, I also investigated the primary preservice teachers' correct and incorrect responses after the restricted time ended. Moreover, I examined the questions that remained unanswered even after the restricted time ended. In this section the findings regarding the second sub-research question which was *"What are the performances of preservice primary teachers when solving structurally-related two-digit addition and subtraction problems outside the allocated time?"* will be addressed. The performances based on the given the allocated time will be discussed.

Time management could be associated with one of the strands of mathematical proficiency, which was procedural fluency and was defined as the ability to perform procedures flexibly, accurately, efficiently and appropriately (NRC, 2001). Preservice primary teachers in this study seemed to be not completely fluent since 9.1% of the overall answers were given outside the time or stayed unanswered in the given time although there were shortcuts of the problems. This study revealed that one of the main reasons for this situation is the dependency on pen-paper and rule-based strategies. Some of the participants emphasized that the need for the materials like paper and pen to solve faster than computing mentally. It was observed that even some of the participants wrote in air with fingers when doing standard algorithm. Participants explained their dependency by saying the need to imagine writing the operation and even erasing it. They stated that they had a hard time doing the operation mentally and if they had paper and pen they would do with them. This dependency on the paper and pen was observed in some studies in the literature (Alsawaie, 2012; Reys & Yang, 1998; Şengül, 2013; Yang, 2005; Yang et al., 2009). As Yang (2005) stated paper and pen strategies and dependency on standard algorithm limits individual's capacity and poses a considerable obstacle for developing number sense. In this study, the participants also chose using standard algorithm strategy the most and they stated that while applying standard algorithm, trading process takes so much time than the given restricted time. The results demonstrated that being dependent on materials like paper and pen, or rule based strategies like standard algorithm was a significant reason why the participants could not compute mentally in restricted time period. Also, as Beishuizen (1993) stated before, even if the participants know that the standard algorithm is not the fastest way, they choose to apply that strategy because they stated that they feel more comfortable with it.

It was seen that some of researchers used restricted time while assessing the mental computation abilities (Peters et al., 2010; Şengül, 2013; Torbeyns et al., 2008). In this study, the preservice primary teachers were given five seconds to answer each problem. When the findings of this study were examined, it was observed that most of the participants had trouble answering within the restricted time while mentally calculating. 5.4% of the problems were answered correctly or incorrectly outside of the given time. Also, the participants could not give any answers to 3.7% of the problems even after the given time was over. Notably, seniors had the highest number of incorrect answers after time ended in every subcategory except far complement. Moreover, they were the ones who had the least number of unanswered problems. This indicated that the senior participants had a greater tendency to answer problems than other participants. On the other hand, sophomores had the lowest number of incorrect answers after time ended in almost every subcategory. However, they had the highest number of unanswered problems. Similarly, this could indicate that they had more hesitancy to answer questions than other participants. This situation could be explained by the fact that the senior preservice primary teachers took the mathematics teaching courses and the sophomores did not take these courses when the data were collected.

The answers that were given after time ended was examined according to the categories. The data collection problems consisted of three different categories. The first category was decoy, far complement and near complement problems. Considering the first category of this study and the fluency of the participants, decoy category had the most unanswered questions after the restricted given time ended. Similar with

Paliwal and Baroody (2020), the shortcut problems were not applicable for solving decoy problems, and their results showed that in these decoy problems, the students who participated in the study were not faster problem solvers in comparison with near and far complement problems. In addition, decoy problems had the greatest number of answers after the given time ended in category one among freshmen, juniors and seniors. Different from that, far complement problems had the highest number that were answered after the restricted time ended among sophomores. On the other hand, another category was the second category which contained small and large distance problems. Peters et al. (2010) found that large distance problems (e.g., 71-2=?) should be answered more quickly. In regard to the category two, the findings in terms of after the restricted time ended were reviewed. The investigations revealed that small distance problems were answered significantly more times than large distance problems after the five seconds expired in the overall results. Besides, when the results were explored across the different years of the participants, the similar findings were observed, and it emerged that large distance problems had fewer number of answers than small distance problems after the time ended. Furthermore, considering the category three, which included addition and subtraction problems, this study’s findings showed no consistency between the operation types in terms of answering within the given time effectively. In the overall results, the number of answers in addition and subtraction problems was close to one another, but addition problems had slightly more answers than the subtraction problem after the restricted time ended. This situation continued with the freshman preservice primary teachers. Among sophomores and seniors, the addition problems had a higher number of answers after the time ended, on the other hand, juniors had a higher number of answers for subtraction problems than addition problems. This result was also persistent with Peters et al. (2010) since they did not find a significant difference between addition and subtraction problems in terms of fluency and time management.

5.1.3 The Mental Computation Strategies

Other research questions of this study were given below.

For not manageable problems within the allocated time,

2. What are the strategies of preservice primary teachers produced outside the allocated time when solving structurally-related two-digit addition and subtraction problems?

2.1. How do these strategies differ by year in the primary education program (i.e., freshman, sophomore, junior and senior)?

2.2. How do these strategies differ by the characteristics of the structurally-related two-digit addition and subtraction problems?

To answer these research question, I asked primary preservice teachers' problem-solving processes and mental computation strategies in the problems where they answered incorrectly (i.e. the problems which they could not answer correctly within the given restricted time) and analyzed those strategies. In this section the mental computation strategies of preservice primary teachers while solving part part whole related two-digit addition and subtraction problems will be addressed and discussed according to the level of the teacher education program and the characteristics of the problems.

The preservice primary teachers used number of different mental computation strategies while making mental computations. This research showed that while freshman preservice primary teachers used 16 different mental computation strategies, sophomore preservice primary teachers used 18 different strategies. On the other hand, juniors and seniors both used 22 mental computation strategies. According to these findings, the number of different strategies increased as the preservice teachers' level increased, and these strategies varied from standard algorithm to counting strategies. However, standard algorithm was the dominant strategy throughout the findings of this research since 33.9% of the incorrectly answered problems were solved with standard algorithm in overall results. This score was 26.1% in freshman preservice primary teachers, 50.2% in sophomore preservice primary teachers, 29.5% in junior preservice primary teachers and, 27.4% in senior preservice primary teachers. Besides standard algorithm, some of the most used strategies were benchmark, compensation, change both numbers and part part whole strategies.

Number sense was defined as a person's basic comprehension of numbers and operations, as well as their ability and tendency to apply that understanding in a variety of ways to make mathematical judgments and devise helpful strategies for dealing with numbers and operations (McIntosh et al., 1992). The components of number sense were framed as numbers, operations and the application of operations with numbers. Therefore, using a mental number line, decomposing and recomposing numbers, using benchmarks, flexible mental computation, understanding problems, inventing different strategies and applying those strategies could be the components of the number sense (Berch, 2000; Jordan et al., 2006; McIntosh et al., 1992; Reys et al., 1999). Mental computation was an aspect of number sense and was defined as calculating without an external instrument. In this current study, some of the participants expressed their need for paper and pencil while mentally calculating. Since using paper and pen strategies severely restricts an individual's potential and makes it difficult for them to develop number sense, this study's findings showed that being dependent on pen and paper cannot be overlooked and that it is literally the opposite of mental computation (Yang, 2005).

There were three categories of problems with data collection: decoy, far complement and near complement as category one; small distance and large distance as category two; subtraction and addition as category three. Therefore, the mental computation strategies of preservice primary teachers were analyzed accordingly. In this matter, Paliwal and Baroody (2020) claimed that subtraction as addition strategy could be used in near complement problems, however this strategy has to be used in far complement problems. When this study's findings of the overall number of strategies were considered, there were indications that participants used subtraction as addition in near complement problems, yet not all of the participants used subtraction as addition in far complement problems. In fact, it has been seen that the standard algorithm was the most used strategy in far complement problems, although the participants were given shortcuts to apply subtraction as addition strategy. Nevertheless, the fact that subtraction as addition strategy was the second most used strategy in far complement problems somehow supports the study of Paliwal and Baroody's (2020). Differently, since the shortcut does not apply to solving decoy problems, the participants were not expected to complete these tasks by using subtraction as addition strategy in Paliwal and Baroody (2020) study. Likewise, when this current study's findings of overall mental strategies were investigated, it was also seen that subtraction as addition strategy was one of the least used strategy in decoy problems. Furthermore, standard algorithm was the most used strategy in these problems throughout all of different levels except freshmen. The freshman preservice teachers used partitioning strategy the most and they stated that this strategy seems like a shortcut for them.

According to the findings of this research, participants perceived standard algorithm is the "normal addition", "normal subtraction", "direct method", "what is taught at school" and "comfortable way". They stated that they are comfortable with the standard algorithm and it is easier method for them. In the literature, there were studies in which the structure of the problems was especially designed according to the use of specific strategies, and similar with the current study's results, these studies observed that the participants used the strategies they felt comfortable regardless of the problem characteristics (Beishuizen, 1997; Blöte et al., 2000; Güç & Karadeniz, 2016; Torbeyns & Verschaffel, 2015). Beishuizen (1993) emphasized that although it was argued that there could be best strategy for some problems, students could choose their strategy according to what is suitable for them. The findings of this current research confirmed this argument since the participants had a tendency to use standard algorithm. However, even if the preservice primary teachers felt comfortable with standard algorithm, they could not perform successfully and find the correct answer with that strategy.

When the findings of this study were examined, the mental number line, which was one of the components of the number sense, was observed but very rarely. In addition, using benchmarks, using unexpected strategies and applying them show that some preservice primary teachers had a few of the number sense components. Considering the findings, it was seen that numerous strategies were utilized and some participants described their solution process with more than one strategy. These flexible switching between one strategy to another could be also an indicator for existence of number sense. Switching between possible different representations was stated as another essential component of the number sense (Resnick, 1989). Also, the participants of this study described different unexpected strategies. These strategies were listed as Adding Tens, Skip Counting, Making Similar, Reverse Sequencing, Switching, Thinking Symmetrical and Standard Algorithm but Different Order. Unlike the standard algorithm, Standard Algorithm but Different Order strategy was applied by computing tens first, then moving on to the ones. The participants were prone to make mistakes by using this way since they did not accurately respond with this strategy.

As could be seen in the findings, some of the preservice primary teachers had some indicators of number sense by using some unexpected strategies. Moreover, as stated by several different researchers, inventing different strategies was also an essential indicator of number sense (Berch, 2000; Jordan et al., 2006; McIntosh et al., 1992; Reys et al., 1999). Although they had used some unexpected strategies, these strategies were not invented strategies and also the strategies were ineffective for mental computation when the restricted given time is present. Also, we could understand from the above strategies that the standard algorithm but different order strategy was not an accurate, effective, flexible strategy and even caused some errors because this strategy came from a misconception in the first place. In addition to that, the counting strategies were also apparent in this study, such as counting up from, counting back to, counting on from larger and counting back from. In fact, even counting on from smaller was one of the strategies that was used explicitly by some participants. It could be deduced that these participants who used counting strategies stayed in developing early number sense phase (Sood & Mackey, 2015).

This study revealed that the standard algorithm was the most used strategy among all levels of preservice primary teachers in all categories except only decoy problems freshman preservice teachers used partitioning strategy the most. Considering the findings of this research that showed the incorrect and unanswered problems' strategies, it could be inferred that those who use standard algorithm when mentally computing have a tendency to make mistakes. However, these results were fairly different from some studies in the literature where the standard algorithm was the dominant strategy but employed successfully and accurately (Carroll, 2000; Torbeyns & Verschaffel, 2013; Torbeyns & Verschaffel, 2016). Differently, in the present study, the preservice primary teachers specifically had difficulty with keeping the borrowed amount in their mind such as some of the participants stated that they either added one extra ten or forgot the traded number. The findings of this research were similar to those of the related studies in which the standard algorithm was determined to be the most used strategy and the participants showed a reliance on rule based and school taught strategies and had poor mental computation abilities (Alsawaie, 2012; Kabaran & Işık-Tertemiz, 2019; Kayhan-Altay, 2010; Reys et al., 1999; Reys & Yang, 1998, Şengül, 2013; Şengül et al., 2012; Yang, 2005; Yang & Huang, 2014; Yang et al., 2009). The preservice primary teachers chose to use the standard algorithm the most, even with the large distance problems like 28+3=31, 31-28=?. They knew and declared that these large distance problems were not difficult, however they still preferred to find the answer with the standard algorithm. In large distance problems, it was seen that the standard algorithm was used 102 times in total.

In some cases, participants described their thinking as trying to use strategies like compensation other than the standard algorithm, but they got confused, tried to remember the next step, and eventually returned to use standard algorithm because they did not conceptually understand addition and subtraction with these strategies. Like Bums (1994) declared that learning traditional algorithms caused one to perceive mathematics as a collection of rules and steps that could be memorized. This situation could be explained by not internalizing other strategies, and preferring standard algorithm strategy like rule based strategies to just automatically applying them. Conceptual understanding was another strand of mathematical proficiency and it was about comprehending mathematical concepts, operations and relationships (NRC, 2001; Van de Walle et al., 2013). Since with compensation, they still did not conceptually understand, they just tried to remember what is the next step and could not decide whether they should add or subtract the number they manipulated. They could not assess the compensated parts and whole of the operation and the relations among those numbers. Therefore, as stated before by Blöte et al. (2000), this study also supported that the conceptual understanding of the participants is related to flexibility and strategy choice.

The number sense of the preservice primary teachers was assessed in light of the findings. Number sense was seen as the ability to relate part and whole relations (Olkun & Toluk-Uçar, 2018). Additionally, fact families were the number facts that directly related to parts and whole in the problem structure, and it was stated that fact families could help students understand the complementary relations between operations (Cobb, 1987; Sun & Zhang, 2001; Zhou & Peverly, 2005). In this matter, operation sense was another ability and individuals with operation sense could comprehend the invertibility of operations and could build the connections among operations (Slavit, 1999). Examining the findings of this research showed that preservice primary teachers seemed to not recognize the part part whole relations and fact families in an operation, conceptually understand the problems, apply strategies according to the characteristics of the problem, and have fluent and flexible mental computation skills. They not only had inadequate number sense but also they did not gain operation sense to develop relations between numbers and operations.

Considering mathematical proficiency and the strands of it, preservice primary teachers had insufficient conceptual understanding, procedural fluency and strategic competence. Since mathematically proficient people have relevant mathematical knowledge and are able to use this knowledge in appropriate situations, the participants of this study did not show indications by not using shortcuts and invented strategies. It could be inferred that they have not become mathematically proficient in the strands of conceptual understanding, procedural fluency and strategic competence. Also, NRC (2001) explained how all five aspects of mathematical skills were interconnected and increased together. The results of the study could confirm this to some extent because they had limited abilities on all of these strands. Moreover, Pope and Mangram (2015) expressed that mathematical proficiency and number sense were connected. As analyzed before, the number sense development of these participants was not sufficient, like their abilities for mathematical proficiency. This study's findings were also consistent with the researchers on the literature as having an insufficient number sense similar with the studies carried out in different parts of the world with different age groups, being dependent on paper and pencil and rule based strategies such as standard algorithm, not selecting appropriate strategies in accordance with the given problem characteristics and inventing strategies which do not serve as a flexible tool for mental computation (Alsawaie, 2012; Beishuizen et al., 1997; Carroll, 2000; Güç & Karadeniz, 2016; Kabaran & Işık-Tertemiz, 2019; Kayhan-Altay, 2010; Reys et al., 1999; Reys & Yang, 1998, Şengül, 2013; Torbeyns et al., 2008; Torbeyns & Verschaffel, 2015; Yaman, 2015; Yang, 2005; Yang & Huang, 2014; Yang et al, 2009). The results of this research were particularly concerning as the participants are preservice primary teachers that will educate students in the future.

5.2 Limitations, Recommendations and Implications

This study aimed to investigate the mental strategies of preservice primary teachers while computing two-digit addition and subtraction problems that are related in terms of the part part whole structure. The previous section provided a detailed overview of the study's findings. There are also some limitations of this qualitative study. This section discusses the study's limitations, recommendations for further research, and educational implications.

Firstly, some researchers found that participants did not always utilize the strategy they claimed and they suggested that the verbal data may not always accurately reflect the strategy that was actually used (Torbeyns et al., 2009; De Smedt et al., 2010). This situation could be present in this study but participants' statements are taken into account.

Secondly, there was a limited time in one to one interviews with preservice primary teachers. More time could be invested to get richer data from the participants' problem-solving processes. Therefore, much extensive interviews could be conducted. In addition to the interviews with the addition and subtraction problems, the number sense test could also be implemented to compare the used strategies and their number sense level. After implementing a number sense test and determining the levels of students and this study's results could be analyzed accordingly.

Thirdly, this research was carried out with 86 preservice primary teachers randomly selected from among the primary school teacher candidates in a state university that accepted students with one of the highest university entrance exam scores. This study could be conducted on the different universities from the different regions of the Turkiye so that it could give more information about preservice primary teachers. Also, a longitudinal study could be conducted, and the same group of participants' number sense and mental computation abilities could be investigated throughout different years of the primary education program.

Regarding the educational implications, a primary teacher candidate graduates with a total of 240 ECTS and the place that mathematics teaching takes from the program is about 5.4%. This situation is problematic and the more emphasis should be placed on teaching mathematics and mathematics related courses in primary teacher education. Furthermore, the primary teacher educators' number sense and mental computation abilities could be investigated, and the competencies of opening courses related to these abilities could be studied. Moreover, the curriculum of primary education programs could be examined to see which universities had what kinds of courses in mathematics education. The content of these courses could be explored. In addition, this study showed different strategies emerged from different problem types. There could be courses where different problem types were discussed.

It should not be denied that the preservice teachers did not come to the education faculty with empty-handed and how they learned mathematics during their school years has effect on their undergraduate educations. Considering the age group of the preservice teachers, it is seen that they could not get rid of the effects of the exam-oriented and rule-based mathematics education in their past lives. Therefore, mental computation and number sense skills should be taught to students starting from primary school ages with not only procedural activities but also with conceptual development supporting activities. For this, teachers should have the knowledge and skills that will enable their students to gain number sense and mental computation skills.

To develop students' number sense for a better-quality mathematics education, teachers' number sense should be developed first (Yang et al., 2009). Therefore, the first thing to do is to provide teachers with the knowledge of learning and teaching number sense. These changes should cover preschool, primary and elementary mathematics teaching. The future studies should be aimed at improving the existing situation rather than detecting it. How different learning environments, such as activity based or technology supported, affect the development of students' number sense is one of the topics that need to be investigated. In addition, preservice and in-service teachers can take training about number sense, and this trainings effects could be examined. Since this study showed different problem types and the relationship between the problem types and mental computation strategies, professional development could be given to the in-service teachers regarding problem types. Moreover, the effects of this development could also be investigated.

Finally, Baroody & Paliwal (2020) conducted their study with children and the results was inconsistent in terms of using shortcut operation and applying subtraction as addition strategy. However, this current study integrated Peters et al.’s (2010) data collection tool and presented a new data collection tool. Therefore, I could also recommend conducting a research with elementary school children with this new data collection tool.

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APPENDICES

A. Ethical Permission from Middle East Technical University

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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Sayı: 28620816/ 01 ARALIK 2021

Konu : Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi : İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr.Öğr.Üye Şerife SEVİNÇ

Danışmanlığını yürüttüğünüz Ecem ÇELİKKOL'un "Sınıf Öğretmenliği Öğretmen Adaylarının Toplama-Çıkarma İşlemlerini İlişkilendirme Süreçlerinin İncelenmesi" başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **453-ODTU-2021** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Prof.Dr. Mine MISIRLISOY
İAEK Başkanı

B. Permission from the University Where the Data Was Collected

Evrak Tarihi ve Sayısı: 31.01.2022-E.278530

T.C.
GAZİ ÜNİVERSİTESİ
GAZİ EĞİTİM FAKÜLTESİ
Temel Eğitim Bölüm Başkanlığı

Sayı : E-98037648-044-278530 31.01.2022
Konu : Anketler (Ecem ÇELİKKOL)

GAZİ EĞİTİM FAKÜLTESİ DEKANLIĞINA

İlgi : 31.01.2022 tarihli ve 89377925-044- 277638 sayılı yazı.

Orta Doğu Teknik Üniversitesi Fen Bilimleri Enstitüsü Matematik ve Fen Bilimleri Eğitimi Ana Bilim Dalı Matematik Eğitimi Bilim Dalı yüksek lisans programı öğrencisi Ecem ÇELİKKOL'un Dr. Öğr. Üyesi Şerife SEVİNÇ danışmanlığında yürüttüğü "Sınıf Öğretmenliği Öğretmen Adaylarının Toplama Çıkarma İşlemlerini İlişkilendirme Süreçlerinin İncelenmesi" başlıklı tez çalışması kapsamında uygulama yapma talebi Başkanlığımızca uygun görülmüştür.

Bilgilerinize ve gereğini arz ederim.

Prof. Dr. Rabia SARIKAYA
Bölüm Başkanı

Bu belge, güvenli elektronik imza ile imzalanmıştır.
Belge Doğrulama Kodu :BSCCT99PK3 Belge Takip Adresi : https://belgedogrulama.gazi.edu.tr/belgedogrulama.aspx

Gazi Üniversitesi Gazi Eğitim Fakültesi Temel Eğitim Bölümü P.K.06500 Bilgi için :Türkan Alçakır
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Kep Adresi: gaziuniversitesi@hs01.kep.tr

C. Answer Key Checklist

Answer Key Checklist

no		PST1	PST2	PST3	PST4	PST5	PST6	PST7	PST8	PST10	PST11	PST12
1	31											
2	34											
3	-4											
4	-2											
5	86											
6	87											
7	32											
8	43											
9	-25											
10	-24											
11	122											
12	136											
13	26											
14	37											
15	-44											
16	-67											
17	9											
18	4											
19	25											
20	26											
21	-3											
22	-7											
23	38											
24	44											
25	31											
26	34											
27	-37											
28	-48											
29	162											
30	181											
31	32											
32	43											
33	-26											
34	-29											
35	130											
36	140											
37	3											
38	8											
39	67											
40	59											
41	79											
42	88											
43	18											
44	25											
45	3											
46	7											
47	46											
48	48											

D. Sample Volunteer Participation Form

ARAŞTIRMAYA GÖNÜLLÜ KATILIM FORMU

Bu araştırma, ODTÜ Matematik Eğitimi Bölümü Yüksek Lisans öğrencisi Ecem Çelikkol tarafından Dr. Öğr. Üyesi Şerife Sevinç danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir. Bu form, sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?

Bu çalışmanın amacı, ilköğretim seviyesindeki öğrencileri eğitecek olan sınıf öğretmenliği öğretmen adaylarının çıkarma ve toplama işlemleri arasındaki ilişkiyi anlamlandırma süreçlerini öğrenmektir. Araştırmaya katılmayı kabul ederseniz, sizden beklenen, araştırmacı tarafından tarafınıza sorulacak olan soruları verilen süre içerisinde yanıtlamanızdır. Bu çalışmaya katılım ortalama olarak 20-30 dakika sürmektedir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?

Araştırmaya katılmayı kabul ederseniz, size gösterilecek olan matematik sorularını belirtilen süre zarfında yanıtlamanız ve cevapları bulma süreciniz hakkında açıklama yapmanız istenecektir.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız?

Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Ankette, sizden kimlik veya kurum belirleyici hiçbir bilgi istenmemektedir. Görüşmeler kayıt altına alınacaktır. Cevaplarınız tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecektir. Katılımcılardan elde edilecek bilgiler toplu halde değerlendirilecek ve bilimsel yayımlarda kullanılacaktır. Sağladığınız veriler gönüllü katılım formlarında toplanan kimlik bilgileri ile eşleştirilmeyecektir.

Katılımınızla ilgili bilmeniz gerekenler:

Bu çalışma kişisel rahatsızlık verecek sorular veya uygulamalar içermemektedir. Katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz cevaplama işini yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye, çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır. Çalışma sonunda, bu araştırmayla ilgili sorularınız cevaplanacaktır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz: Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Araştırma hakkında daha fazla bilgi almak için Matematik ve Fen bilimleri Eğitimi Bölümü öğretim üyelerinden Dr. Öğr. Üyesi Şerife Sevinç (E-Posta: sserife@metu.edu.tr) ya da yüksek lisans öğrencisi Ecem Çelikkol (E-posta: celikkolecem@gmail.com) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.

(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyisim	Tarih	İmza
	----/----/-----	